

The Ruler Matters: Factor Exposures, Attribution, and Portfolio Risk

Mingyang Liu

Department of Finance, Imperial Business School
Imperial College London

yang.liu19@imperial.ac.uk

October 2026

Abstract

Firm characteristics parameterize conditional factor loadings in IPCA. A larger characteristic library adds new directions while also including directions that overlap with those already represented. We use the cross-sectional distribution of firm characteristics as a ruler to evaluate fixed-dimensional IPCA loading spaces. Holding the number of factors fixed, expanding from 36 to 153 characteristics can improve in-sample fit while representing momentum and size positions less fully. Reallocating shared factor exposures changes their attribution without changing joint fitted returns. Holding model coefficients fixed and the characteristic target constant, changing the exposure reference changes stock holdings and trading demands, as well as subsequent return risk. The findings show that factor models should be evaluated by the economic exposures they represent and the portfolio decisions those exposures support, alongside return fit.

Keywords: Conditional factor models; firm characteristics; characteristic exposures; portfolio holdings; portfolio risk.

JEL Classification: G11, G12, C38, C52.

I Introduction

Instrumented principal component analysis (IPCA) uses firm characteristics to describe how stocks load on common return factors (Kelly et al., 2019). A richer set of characteristics may improve the fit of a factor model, but does it preserve the stock exposures represented by its estimated loadings? Some additional characteristics distinguish firms along genuinely new directions; others describe much the same relative positions as existing characteristics. When the number of factors is fixed, the model reestimates a low-dimensional loading space from this broader set of candidates. The new space need not contain the one estimated from the smaller library, even though the characteristics themselves are nested. Thus, a better-fitting model may represent a familiar momentum or size position less fully. The question is which stock exposures the model represents, not whether adding more variables must mechanically dilute every existing characteristic.

This issue is not limited to changes in the estimated model. Conditional loadings depend on firms' characteristics, which evolve as firms change and the investable universe turns over. Even if the coefficients mapping characteristics into factor loadings are held fixed, the corresponding loading profiles across stocks, and their overlap with a particular characteristic position, can change. These two sources of variation—reestimation with a larger information set and changes in the stock cross section under fixed coefficients—matter for different but connected decisions. Researchers use factor models to attribute fitted returns to observed and latent components. Investors use estimated exposures to manage risks while seeking to preserve a desired characteristic position. Neither task can be understood from estimated coefficients or return fit alone: what matters is which stocks carry the implied exposures. We study how an expanding characteristic library changes that representation, how overlapping stock profiles affect reported attribution, and how historical versus current exposure references affect the holdings and risks of portfolios with the same characteristic objective.

Two approaches to specifying common factors are familiar in empirical asset pricing. The first uses *observable factors*: pre-specified return series, commonly constructed from traded portfolios. The market excess return and the size, value, profitability, investment, and momentum factors constitute our six-factor benchmark (FFC6) (Fama and French, 1993, 2015; Carhart, 1997). These portfolios provide economically recognizable benchmarks for explaining stock returns and evaluating factor exposures. Their labels identify the portfolio construction rules, not necessarily the fundamental shocks underlying risk compensation. The second approach estimates *latent factors* from return data. These are statistical common-return components whose time series need not be

specified or economically named in advance. They can capture comovement beyond the chosen observable benchmarks, although such comovement need not correspond to a distinct priced fundamental risk. Throughout the paper, *latent factors* refers to statistically estimated common-return factors, rather than directly observed economic shocks (Kelly et al., 2019; Kozak et al., 2018).

IPCA brings these approaches together by using observable characteristics to parameterize conditional loadings while jointly estimating latent factor returns. We consider a specification that also includes pre-specified observed factor returns and a characteristic-associated intercept:

$$r_{i,t+1} = \alpha_{i,t} + \delta'_{i,t} g_{t+1} + \beta'_{i,t} f_{t+1} + \varepsilon_{i,t+1}, \quad (1)$$

where g_{t+1} denotes the observed factors, f_{t+1} the latent statistical factors, and $\delta_{i,t}$ and $\beta_{i,t}$ their conditional loadings. The intercept $\alpha_{i,t}$ accommodates fitted average returns not attributed to the included factor components. The distinction between factor coordinates and factor spaces is essential here. Latent factor returns and their individual loading vectors depend on the normalization used to resolve rotation indeterminacy; an equivalent change of factor basis leaves their joint fitted contribution and the span of their stock-level loadings unchanged. Our object of interest is therefore the *estimated loading space*, not the economic identity of an arbitrarily numbered latent factor (Kelly et al., 2019; Giglio and Xiu, 2021).

That distinction gives the loading space an economic role without assigning structural names to statistical factors. For researchers, the question is what latent factors add beyond FFC6 and how fitted returns are attributed among observed factors, latent factors, and the characteristic intercept. If these components imply overlapping stock profiles, the allocation of their shared exposures can affect reported contributions even when the relevant combined fitted return is held fixed. Such attribution choices are distinct from merely rotating the latent factors within their own span, which leaves their aggregate fitted component unchanged. For investors, an analogous overlap arises when a manager seeks to retain a momentum or value tilt while limiting unwanted model-implied factor exposures. Neutralizing the entire estimated loading space can also remove part of the intended characteristic exposure; restoring the original target may require different stock weights, gross holdings, and trading. Strict loading neutrality is a useful controlled benchmark, not a prescription to hedge every statistical factor or a guarantee against all common return risk. Both applications raise the same empirical question: how much of a given stock-characteristic position lies in the estimated loading space?

I.A Our Approach

We answer this question by evaluating coefficient directions through the stock profiles they generate. Consider book-to-market and earnings-to-price. Although they are distinct valuation characteristics, they may rank many of the same firms similarly. Coefficient vectors that look different can therefore imply overlapping stock profiles; conversely, a small change in coefficients can have a large effect when stocks differ sharply along the relevant characteristic direction. Let Z_t be the characteristic matrix for the n_t available stocks. Coefficient directions a and b imply the stock profiles $Z_t a$ and $Z_t b$, whose cross-sectional inner product is

$$\frac{1}{n_t} (Z_t a)' (Z_t b) = a' Q_t b, \quad Q_t = \frac{Z_t' Z_t}{n_t}. \quad (2)$$

We refer to this empirical cross-sectional second moment as the *stock-exposure ruler*. It measures overlap in the stocks represented by coefficient directions, rather than treating coefficient-space distances as if they had a uniform meaning across firms. In particular, $a'b = 0$ need not imply $a'Q_t b = 0$: orthogonality of characteristic coefficients need not produce orthogonal stock profiles. The distinction is economically consequential whenever an estimated loading is used to restrict a portfolio. A coefficient-space projection can appear to remove a loading while leaving exposure to its actual stock profile. The relevant comparison is therefore not IPCA estimation “with” or “without” Q_t : characteristic second moments already enter IPCA. Our use of the existing Q_t evaluates the economic content of estimated loadings and the stock positions implied by exposure restrictions. It neither introduces a new factor estimator nor substitutes for the covariance matrix of stock returns.

A useful property of this comparison is that it respects the rotation indeterminacy of latent factors. A nonsingular transformation of the latent factor coordinates changes individual loading vectors but not their span across stocks. Consequently, the projection of a characteristic profile onto the full estimated loading space, and the condition that a portfolio be neutral to every loading in that space, are invariant to an equivalent change of latent-factor basis. These statements concern alternative representations of the *same* estimated space. They do not imply that loading spaces separately estimated with different characteristic libraries coincide, or that a fixed coefficient mapping produces the same stock exposures at different dates. This is precisely the distinction our empirical comparisons investigate.

Our first comparison varies the characteristic information set used to estimate the factor-loading space. We begin with a fixed subset of 36 characteristics and expand to the full 153-characteristic library of Jensen et al. (2023).¹ Across specifications, we hold fixed the FFC6 observed factor returns and the number of additional latent factors. We estimate a model with an unrestricted characteristic intercept and, separately, a model with the intercept constrained to zero. The comparison asks which familiar characteristic stock profiles the separately reestimated loading spaces represent. Because those spaces are not required to be nested, the projection share of a given characteristic can rise, fall, or remain unchanged. We therefore treat the direction and magnitude of each change as empirical outcomes, not as a mechanical allocation of a fixed share across a larger number of features.

Our second comparison keeps the estimated characteristic-to-loading coefficients fixed while changing the cross-sectional reference for measuring exposure. Coefficients are estimated each March and held through the following February; characteristics, available stocks, and portfolio weights are updated monthly. A loading direction that overlaps with momentum in March can overlap differently with the momentum position formed in October. We therefore compare an annual-origin rule using the March reference with a current-reference rule using the contemporaneous cross section. Both policies use the same current stocks and preserve the same original positive characteristic target in each month, but they impose different current loading-neutrality restrictions. This comparison evaluates the holdings and subsequent risk of two complete exposure-control policies; it does not presume that characteristic overlap moves in any particular direction over time.

I.B Findings

We evaluate the fitted models along three distinct dimensions: their fit to realized stock returns, the pricing error of the attributed factor component, and the familiar characteristic stock profiles represented by the estimated loading space. The three assessments need not agree. In the historical descriptive comparison, expanding the library from 36 to 153 characteristics raises total stock-return R^2 from 17.01% to 17.70% in the unrestricted three-latent-factor model, while the relative

¹The 36-characteristic baseline is motivated by the size of the original IPCA characteristic set in Kelly et al. (2019), rather than an attempt to reproduce its exact variables. Kelly, Pruitt, and Su (2019) use stock-level characteristics from Freyberger, Neuhierl, and Weber (2017), whereas the library in Jensen et al. (2023) is constructed separately, drawing primarily on the anomaly compilation of Hou, Xue, and Zhang (2020). Although some characteristics have similar economic interpretations, their definitions and implementations need not coincide. Our 36-characteristic baseline is drawn from the same 153-characteristic universe, ensuring that the comparison is nested within a common data source.

pricing error of the coefficient-attributed factor component increases from 76.84% to 92.08%. When the zero-intercept restriction is instead imposed and the model jointly reestimated, the factor-component relative pricing error declines from 7.25% to 3.55%. The restricted and unrestricted results concern different fitted models, not two allocations of an unchanged fit. Better total return fit therefore does not determine either factor-component pricing or the economic interpretation of the estimated loadings.

The stock-exposure ruler then shows what the reestimated spaces represent. The effective number of directions in the characteristic stock profiles rises from 10.22 to 21.08, indicating that the larger library adds distinct directions alongside overlapping information. In the separate rolling exposure comparison, both libraries are evaluated against the same named stock profiles and training-period ruler at each annual origin. Under the unrestricted specification, the average share of the squared momentum profile represented by the joint loading space falls from 82.27% to 62.33%; for larger-firm size it falls from 90.17% to 71.93%. The pattern is not uniform: coverage of intangibles and firm attributes rises from 35.62% to 37.59%. These shares are squared projections of individual stock profiles, not portions of a common 100% budget distributed across characteristics. Their changes are therefore not dictated by feature count. Nor are they explained return variance or tests of whether these exposures command risk premia. The historical return-fit and rolling stock-exposure calculations have different evaluation windows; together they show why a fit statistic cannot, by itself, establish which familiar positions the estimated loading space represents.

Overlap across stock-loading profiles can change the economic interpretation of a factor model even when its fitted returns remain unchanged. For a larger-firm-size position, reallocating exposure shared by observed and latent factors raises the contribution attributed to observed factors from 1.38 to 45.53 basis points per month. The latent component offsets this increase, leaving their combined fitted contribution unchanged at 2.16 basis points. The same fitted factor payoff can therefore support sharply different assessments of how much of a familiar size position is explained by observable benchmark factors rather than latent components. A similar reallocation between the factor component and the characteristic-associated intercept changes their reported contributions without altering fitted stock returns. These results show that the apparent explanatory role of familiar factors can depend on how shared stock exposures are assigned, rather than on additional return variation explained by the model.

A direct comparison shows why the ruler matters even with the factor model held fixed. In the unrestricted 153-characteristic book-to-market exercise, a projection based only on coefficient

orthogonality leaves, on average, 46.10% of the squared stock-weight norm in the estimated stock-loading space. Projection using the current stock-exposure ruler removes that loading-space component to numerical precision. This is a difference in achieved stock-loading neutrality, not a 46.10% reduction in return variance. Unlike the training-reference coverage statistics above, this comparison evaluates the stock positions at formation. It illustrates why the inner product used to interpret the loadings matters for an investor who acts on them.

Portfolio implementation then makes the same overlap consequential for a fixed investment objective. We construct cash-neutral portfolios that preserve the same original positive book-to-market, momentum, or larger-firm-size target in each month, subject to different estimated loading-neutrality restrictions. Removing the portion of a characteristic position represented by the fitted loading space also removes part of the intended exposure. Restoring that target changes the scale and composition of stock positions and can affect gross holdings, trading, and subsequent risk. For unrestricted larger-firm size, expanding the library reduces mean gross holdings from 3.27 to 1.92 times NAV and monthly volatility from 275.30 to 152.12 basis points under the specified loading-neutral rule. The risk effects are nevertheless heterogeneous: only three of the twelve simultaneous 95% confidence intervals for library-risk comparisons exclude zero. For the specified projection direction, an exact identity relates required gross holdings to loading-space overlap and the distribution of residual stock weights. This characterizes the holdings required by that rule, not minimum gross across all feasible portfolios or the variance of their subsequent returns.

The changing cross-sectional ruler has a further implication even when estimated coefficients are fixed. The annual-origin and current-reference portfolios hold positions in the same current stocks and meet the same original positive characteristic target, but they impose different contemporaneous loading restrictions. Across twelve exploratory positive-target comparisons, current-reference portfolios have 26.62%–69.39% lower estimated subsequent return variance than annual-origin portfolios; nine simultaneous 95% confidence intervals exclude zero. The signed-target family is evaluated separately, and neither mean returns nor turnover improve uniformly. These findings compare complete portfolio policies, not the causal effect of changing only the reference date while keeping current loading constraints identical. Nor do they imply that overlap must rise in volatile markets or that changes in overlap forecast future risk.

I.C Literature

Our analysis relates first to research on characteristic information, conditional factor estimation, and return prediction. Green et al. (2017) ask which characteristics contain independent return information, and Feng et al. (2020) test the incremental pricing content of proposed factors. Kozak et al. (2020) study the stochastic discount factor representation of return predictors, while Gu et al. (2020) and Kelly et al. (2024) investigate richer prediction models. IPCA and subsequent methods use firm characteristics to estimate conditional factor loadings and latent returns (Kelly et al., 2019; Gu et al., 2021; Lettau and Pelger, 2020; Giglio and Xiu, 2021; Zaffaroni, 2025), with applications beyond equity returns (Goyal and Saretto, 2025). Our contribution concerns the economic interpretation and use of these estimates, not the general observation that a larger predictor set can change an estimated factor model. We measure which familiar stock profiles the loading space represents under a common cross-sectional metric, and connect that representation to the positions needed to implement a specified investment objective. This complements the distinction between arbitrary latent-factor coordinates and their economically relevant span.

A second literature studies the relationship between factor compensation and characteristic-associated pricing errors. Zhang (2024) examines pricing-error restrictions with latent factors and firm characteristics, and Choi and Yuan (2026) develop inference concerning alpha components, orthogonality, and basis dependence. Kozak et al. (2018) distinguish reduced-form factor interpretation from structural economic mechanisms. We complement these studies by measuring how shared stock exposures affect the reported contributions of observed factors, latent statistical factors, and the characteristic intercept within a fixed fitted return. The same measurement then disciplines the loading restrictions used in portfolio construction.

A third literature studies characteristic-based portfolios and the control of common return risks. Brandt et al. (2009) map characteristics into investment weights; Daniel et al. (2020) use return covariances to hedge unpriced common variation in characteristic portfolios; and Kelly et al. (2023a) develop principal portfolios. Work on trading costs and implementability emphasizes that portfolio constraints have real consequences (DeMiguel et al., 2020; Jensen et al., 2026; Kelly et al., 2026). Factor-neutral investing itself is established: Kelly et al. (2019) study conditionally factor-neutral pure-alpha portfolios, and Wang (2025) evaluates zero-beta investments based on IPCA and other statistical factor models. Unlike an allocation that optimizes estimated residual alpha, our experiment preserves a fixed original characteristic objective. It examines how the estimated loading

space and the cross-sectional reference used to measure it determine the stocks required to meet that objective and the risks of the resulting positions.

The paper makes three contributions. First, we use a common stock-exposure metric to assess which familiar characteristic positions are represented by estimated factor loadings, rather than inferring economic content from coefficient orthogonality or return fit alone. Applied to separately reestimated fixed-dimensional loading spaces, this comparison reveals heterogeneous changes in representation as the characteristic library expands; a decline in momentum coverage is an empirical outcome, not mechanical dilution from adding variables. Second, we measure how overlap between stock profiles affects the attribution of fitted returns across observed factors, latent factors, and a characteristic-associated intercept, while distinguishing this attribution problem from latent-factor rotation. Third, we connect the same rotation-invariant measurement to a fixed investment objective, characterizing gross holdings for a specified loading-neutral direction and comparing the holdings, trading, and subsequent risks of portfolio policies based on different characteristic libraries and exposure references. The ruler matters not because it changes IPCA estimation, but because it determines how estimated loadings are assessed across stocks and, when used in portfolio constraints, which positions implement a given characteristic objective.

Section II describes the data and factor-model specifications. Section III distinguishes return fit from factor-component pricing, and Section IV examines characteristic overlap and the stock profiles represented by the estimated loading space. Section V studies fixed-target portfolios, while Section VI compares annual-origin and current-reference exposure measurements. Sections VII–IX examine factor attribution, pricing and forecasting, traded-hedge benchmarks, and partial hedging. Section X considers evolving exposure relationships and monitoring. The appendices provide derivations, further comparisons, and inference details.

II Data, Factor Models, and Evaluation

II.A Stocks and Characteristics

We use U.S. stock characteristics from the library of Jensen et al. (2023) and monthly CRSP returns. The full library contains 153 characteristics covering valuation, profitability, investment, financing, past returns, trading activity, and risk. We compare this library with a fixed subset of 36 characteristics. Both include a constant in the loading specification. Table I describes the samples and characteristic groups, and Appendix IA.13 lists the individual characteristics and identifies the 36-characteristic subset.

Characteristics in formation month t are paired with excess returns in month $t + 1$. We retain CRSP-mapped securities with active trading status and no recorded delisting at formation. Within this status-eligible cross section, each characteristic is ranked, divided by its number of nonmissing observations plus one, and centered at one-half. Ties receive their average rank; missing values receive zero. We then require positive market capitalization. This ordering makes the ranks a function of formation information and keeps the characteristic transformation common across the two libraries.

The initial full-sample comparison uses formation months July 1964 through May 2014, or 599 months. Its purpose is to reproduce the expansion question under matched model specifications and common targets. The portfolio evaluation uses formation months March 2007 through November 2025. We estimate source models each March using the previous 240 months, with the first window covering March 1987 through February 2007; the input history begins in March 1984. The 225 formation months produce 216 common months with the returns needed to evaluate every model and target. All rolling comparisons use this same return calendar, preserving its missing months.

For the portfolio evaluation, a fixed hash of the CRSP security identifier assigns stocks to two groups. We estimate factor-loading directions on the source group and evaluate pricing and factor hedges on characteristic portfolios formed from the target group. A stock keeps its assignment throughout the sample. The groups have disjoint securities, while sharing market shocks and the formation rank definitions. For these pricing and payoff-hedge tests, this design separates the stocks used to learn the factors from those used to assess their usefulness; it does not require the two groups' returns to be independent.

For the rolling evaluation, the 36 test portfolios hold each target stock in proportion to its centered characteristic rank. At formation, we divide the weights by their absolute sum so that each portfolio has unit gross exposure. The historical descriptive comparison instead averages characteristic-weighted returns over stocks with observed subsequent returns, without unit-gross normalization; its weights and observed-outcome population are the same across models. These characteristic-managed portfolios retain the native sign of the characteristic. In particular, firm size takes the larger-firm direction and is not the Fama–French small-minus-big factor. In the rolling portfolio evaluation, subsequent return availability never changes the holdings or their denominator. A portfolio-month with a missing return on a nonzero holding remains unscored, rather than reallocating the missing stock's weight after the outcome is known.

Table I: Characteristic libraries and empirical samples*Panel A: Characteristics by economic group*

Economic group	36-characteristic library	153-characteristic library
Accounting quality	2	17
Composite signals	2	7
Financing and leverage	4	15
Intangibles and firm attributes	1	9
Investment	4	23
Trading liquidity	3	9
Momentum	5	10
Profitability	6	22
Historical return risk	4	18
Seasonality	1	10
Value and payout	4	13

Panel B: Evaluation calendars and formation populations

Experiment or population	Calendar or statistic	Sample size or summary
Historical comparison	July 1964–May 2014	599 formation months
Rolling evaluation	March 2007–November 2025	225 formation months
Common payoff evaluation	Same rolling calendar	216 return months
Model estimation	Each March, 2007–2025	19 origins; 240 prior months
Source stocks per formation month	Mean / minimum / maximum	2250 / 2018 / 2744
Target stocks per formation month	Mean / minimum / maximum	2265 / 2042 / 2735

Panel A counts nonconstant characteristics in fixed descriptive economic groups; both libraries also include a loading constant. For the rolling evaluation, a fixed security-identifier hash permanently assigns stocks to disjoint source and target groups. Formation-stock counts are measured before subsequent-return availability is known. The 36 unit-gross target-stock portfolios are used for rolling pricing and payoff-hedge comparisons; the actual-holdings experiments use source stocks and subsequent returns. The historical descriptive comparison instead uses the same unscaled characteristic-managed targets across models. The return and turnover evaluations use separate common masks of 216 and 219 months, respectively.

II.B Conditional Factor Models

Let $z_{i,t}$ be the column vector containing the constant and firm characteristics for stock i at formation. We estimate the conditional factor model

$$r_{i,t+1} = z'_{i,t}\Gamma\alpha + z'_{i,t}Cg_{t+1} + z'_{i,t}Bf_{t+1} + \varepsilon_{i,t+1}. \quad (3)$$

Here $r_{i,t+1}$ is the stock's excess return, g_{t+1} contains observable factor returns, and f_{t+1} contains latent factors. The coefficient matrices C and B map firm characteristics into their factor loadings. The characteristic intercept $z'_{i,t}\Gamma\alpha$ captures the model's expected-return component outside the selected factor-compensation term. Its interpretation therefore requires the factor specification and the convention separating the components.

Our main specification contains three latent factors and the Fama–French five factors plus momentum, denoted FFC6. The initial comparison also includes a model with five latent factors and no observable factors. For each architecture and library, we estimate both an unrestricted characteristic intercept and a zero-intercept specification. The latter is a separate joint estimation with $\Gamma\alpha = 0$. It allows the factor directions to adjust to the restriction. Throughout, “unrestricted” and “zero intercept” refer to this source-model choice, rather than to the intercept in a later time-series regression of a test portfolio on factor payoffs.

Estimation minimizes average stock-return squared error, assigning equal weight to each month and equal weight to observed stocks within that month. The sample characteristic second moment consequently enters the normal equations. Each initial model uses one moment-based initialization and two fixed random initializations, with the converged solution having the smallest loss retained. We keep the same factor count, target portfolios, sample, and convergence criteria when expanding the library. Appendix IA.2 reports the moment calculations and numerical checks.

For out-of-sample evaluation we use payoffs on portfolios with formation-known holdings associated with the estimated loading directions. Let Z_t collect the source stocks' characteristics, $Q_t = Z'_t Z_t / n_t$, and $x_{t+1} = Z'_t r_{t+1} / n_t$. Each row of Z_t represents a stock and each column a characteristic, including the constant; n_t is the formation stock count. Thus Q_t measures characteristic second moments, while x_{t+1} collects characteristic-managed excess payoffs. The source latent payoff is

$$p_{t+1} = (B'Q_tB)^{-1}B'\{x_{t+1} - Q_tCg_{t+1}\}. \quad (4)$$

The first term is the return on explicit source-stock holdings; the second subtracts positions in the observable factors. The annual B and C are estimated from past data. This construction uses no fitted characteristic-intercept offset. It permits the realized payoff, the estimated risk premium, and the target’s loading to refer to the same financial instrument.

We estimate target regression slopes and intercepts from the annual model’s available calibration history. Each month’s forecast then uses the previous 240 calendar months with complete factor and target observations, requiring at least 120 observations. Factor means and the historical-mean target benchmark use exactly the same dates. This arrangement holds the forecasting information set fixed while allowing factor directions to change at the annual estimation dates. These forecast comparisons supplement the main stock-holdings tests. The following measures first describe the portfolio mandate and its implementation, then define model fit, pricing, and prediction.

We distinguish two forms of evaluation throughout. Pricing and payoff-hedge comparisons use the separately assigned target stocks. The stock-holdings experiments use source stocks but evaluate their subsequent returns, with all portfolio decisions made before those returns occur. The former asks whether learned factor payoffs help explain other securities; the latter asks whether an exposure restriction can be implemented on the stocks described by the model. Reporting both makes the statistical comparison and the investment task explicit.

II.C Performance Measures

Portfolio Holdings and Subsequent Returns

Our primary outcomes describe the stock portfolio implementing a characteristic mandate. For the rolling portfolio comparisons we report the arithmetic mean of subsequent monthly excess returns and their sample standard deviation, using $T - 1$ in the variance denominator, where $T = 216$ is the number of common scored return months. Both are expressed in monthly basis points. The mean measures the modeled excess payoff before execution and security-specific financing costs, while volatility measures its realized return risk. We evaluate all compared policies on the same 216 return months, so different missing-return histories do not create a ranking.

For stock weights w_t per unit of portfolio net asset value, characteristic exposure is $w_t'Z_t a$, where a selects the named characteristic column of Z_t , and gross holdings are $\|w_t\|_1 = \sum_i |w_{i,t}|$. Gross of 2.34 means 2.34 dollars of long and short stock positions in total per dollar of portfolio value. Net stock investment is $\sum_i w_{i,t}$, which is zero in the constrained comparisons. These quantities are measured at formation, before the subsequent return, over all 225 formation months.

An equal-gross comparison may retain different characteristic tilts; a comparison at the same tilt instead lets gross holdings reveal the required scale of the position.

One-way turnover is $\frac{1}{2} \sum_i |w_{i,t} - w_{i,t}^{\text{pre}}|$, where $w_{i,t}^{\text{pre}}$ is the previous stock holding after its realized return and the change in portfolio wealth. The sum covers the union of old and new securities, including entries and exits. We report its mean in percent over the 219 months with the information needed to compute these pretrade holdings. Appendix IA.9 gives the exact wealth adjustment and missing-return convention. Turnover measures trading volume; execution costs, financing terms, and an investor's utility require additional inputs.

We also compare portfolio return variances as $1 - \text{Var}(r^{\text{treatment}}) / \text{Var}(r^{\text{reference}})$, where the two return series are the treatment and reference portfolios' subsequent excess returns. Each exhibit names the reference. The ratio translates a change in volatility into the fraction of the reference variance removed. Ratios with different reference variances are not additive. The following model-fit, pricing, and forecast measures address complementary questions about the estimated factor payoffs and are kept separate from these stock-holdings outcomes.

Model Fit, Pricing, Forecasting, and Factor-Payoff Hedging

For the supporting tests, we evaluate three uses of a factor model: describing common movements in realized returns, explaining differences in average returns, and forecasting returns from past information. These uses need not favor the same specification. Following Kelly et al. (2019) and Kelly et al. (2023b), we report separate measures for each, evaluated on the same test assets and dates.

For the out-of-sample portfolio evaluation, let $y_{j,t+1}$ be the excess payoff of test portfolio j , and let $F_{t+1} = (g'_{t+1}, p'_{t+1})'$ contain the observable factor returns and source latent portfolio payoffs from (4). The target regression estimates slopes $b_{j,t}$ and an intercept $c_{j,t}$ from returns available by portfolio formation. Its factor component is $q_{j,t+1}^F = b'_{j,t} F_{t+1}$, and its complete fitted payoff is $q_{j,t+1}^C = c_{j,t} + q_{j,t+1}^F$. The intercept $c_{j,t}$ belongs to this target regression. The source conditional intercept $z'_{i,t} \Gamma \alpha$ is estimated separately in the stock panel and affects estimation of the source loading directions.

Total R^2 measures the fraction of realized payoff variation described by a specified fit. For $v \in \{F, C\}$, define

$$R_{\text{total},v}^2 = 1 - \frac{\sum_{j,t} (y_{j,t+1} - q_{j,t+1}^v)^2}{\sum_{j,t} y_{j,t+1}^2}. \quad (5)$$

We label the factor-component and complete fits separately. Both use zero excess payoff as the benchmark, without demeaning. Loadings and portfolio weights are determined from past information, while the factor payoffs are realized during the evaluation month. Thus, total R^2 evaluates the model's description of subsequent return variation. Average time-series and cross-section R^2 , defined in Appendix IA.15, show whether this description extends across assets and months.

Average-return pricing asks a different question: does compensation from the factors account for the average payoff of each target? Let J be the number of test portfolios, let T be the number of common evaluation months, and define

$$\begin{aligned}\alpha_j^{\text{PE}} &= T^{-1} \sum_t (y_{j,t+1} - q_{j,t+1}^F), & \bar{y}_j &= T^{-1} \sum_t y_{j,t+1}, \\ \text{RPE} &= \frac{\sum_j (\alpha_j^{\text{PE}})^2}{\sum_j \bar{y}_j^2}, & \text{RMS}_{\text{PE}} &= \left[J^{-1} \sum_j (\alpha_j^{\text{PE}})^2 \right]^{1/2}.\end{aligned}\tag{6}$$

Relative pricing error (RPE) compares squared average model errors with squared average excess payoffs. Lower values indicate a better explanation of average returns. RMS pricing error gives the magnitude in monthly basis points. Excluding $c_{j,t}$ makes these measures evaluate factor compensation; the average residual from the complete target regression is reported separately when relevant.

Return forecasts replace future factor realizations with estimates of factor risk premia available at formation. We estimate those premia by historical factor means, $\hat{\lambda}_t$, and form the factor forecast $m_{j,t}^F = b'_{j,t} \hat{\lambda}_t$ and the complete forecast $m_{j,t}^C = c_{j,t} + m_{j,t}^F$. Predictive R^2 is

$$R_{\text{pred},v}^2 = 1 - \frac{\sum_{j,t} (y_{j,t+1} - m_{j,t}^v)^2}{\sum_{j,t} y_{j,t+1}^2}, \quad v \in \{F, C\}.\tag{7}$$

It measures the improvement in squared forecast error over a zero-payoff forecast. The distinction from total R^2 is economic: factor realizations can describe common return movements that investors could not forecast at formation.

We also compare forecasts with the target's historical mean. This comparison replaces the denominator in (7) with $\sum_{j,t} (y_{j,t+1} - \bar{y}_{j,t}^{\text{past}})^2$, where $\bar{y}_{j,t}^{\text{past}}$ and $\hat{\lambda}_t$ use the same completed past observations. We label it R^2 relative to the historical mean. Reporting both benchmarks reveals whether

a forecast improves on zero, on a simple estimate of the target mean, or on both. Forecast RMSE reports the square root of the average squared forecast error in monthly basis points.

Finally, a factor can be useful as a hedge even when its average return is difficult to forecast. The hedge takes positions $b_{j,t}$ at formation, leaving payoff $h_{j,t+1} = y_{j,t+1} - b'_{j,t}F_{t+1}$. We measure hedge variance reduction by

$$\text{HVR} = 1 - \frac{\sum_j \text{Var}_t(h_{j,t+1})}{\sum_j \text{Var}_t(y_{j,t+1})}. \quad (8)$$

This measure evaluates the aggregate reduction in the individual targets' risks. We also report residual volatility to express the remaining risk in payoff units. The target regression intercept is not a hedge instrument; positions in factor payoffs determine the hedge. Comparisons with familiar characteristic portfolios then establish what the latent factors add to an investor's available hedges.

Paired twelve-month calendar blocks measure uncertainty while preserving common shocks and missing observations. The central portfolio results use three separate families of twelve comparisons: library-expansion volatility differences, current-versus-origin variance reductions at the training rule's signed tilt, and the same variance comparison at the original positive target. Each family uses 19,999 draws and simultaneous 95% intervals; the positive-target family was specified after the training-reference feasibility outcome. Other reported intervals retain their original pointwise interpretation. All inference conditions on the fitted models and observed payoff paths. Appendix IA.8 defines both procedures and their distinct coverage.

III More Characteristics and the Explanation of Average Returns

Better stock-return fit need not give a better account of average returns. Table II holds the sample and 36 characteristic-managed targets fixed as the source library expands. It contrasts how well the fitted stock model describes realized returns with the pricing errors of its factor component. The distinction matters because these two uses of the model reward different features of returns.

The stock-return objective primarily rewards a close description of monthly return realizations, including common fluctuations around their means. Average-return pricing asks whether factor compensation accounts for the targets' mean payoffs. A direction that explains substantial realized variation need not carry a precisely estimated average premium, and a priced direction need not account for much monthly variance. Lettau and Pelger (2020) study the distinction by incorporating both return variation and average returns in factor estimation.

Expanding the library improves stock fit but worsens the unrestricted factor component's pricing error. With three latent factors and FFC6, stock-return fit rises from 17.01% to 17.70%, while relative pricing error rises from 76.84% to 92.08%. Here the characteristic intercept and latent loadings are separated using ordinary coefficient orthogonality. With five latent factors, the pricing error rises from 70.93% to 127.64% even as stock fit improves. A relative pricing error above 100% means that this factor component explains target means less accurately in squared error than a zero-payoff benchmark; it does not mean the portfolio earns a negative return or that the full fitted model has this error.

Figure 1 makes the discrepancy visible. Expanding the library improves realized stock fit under both intercept specifications, but its pricing effect changes when the intercept is restricted. Which part of the fitted average return is assigned to latent factors, rather than the characteristic intercept, is therefore central to interpreting the apparent pricing performance.

The pricing ranking changes when we jointly estimate the model with a zero characteristic intercept. Expansion then reduces relative pricing error from 7.25% to 3.55% in the three-latent-factor model and from 1.42% to 0.44% in the five-latent-factor model. The restriction leaves the factors responsible for mean-return components that the unrestricted fit can allocate to its intercept. Thus, both the model's characteristic library and the source-intercept specification govern the reported factor-pricing comparison.

A jointly estimated zero-intercept model differs from deleting the intercept of an unrestricted fit. Deleting it holds the previously chosen factor directions fixed; joint reestimation lets those directions adjust to the restriction. We use the latter procedure, consistent with the restricted and unrestricted IPCA distinction in Kelly et al. (2019). This is a comparison of separately fitted models, not an algebraic reallocation of one estimate.

We also separate the intercept from factor compensation without refitting the unrestricted model. Projecting the characteristic intercept onto the latent loading space under the empirical ruler and transferring that component to the factor mean preserves the complete stock fit. In the three-latent-factor specification, the reassigned factor component has relative pricing errors of 53.89% with 36 characteristics and 72.90% with 153. The attribution changes substantially, but the larger library still has the higher error. This exercise changes the accounting of a fixed fit, rather than the fitted stock returns themselves.

The intercept and latent factors can place exposure on the same stocks, so different decompositions of their shared direction can leave the complete model unchanged. A shift in the latent-factor

Table II: Characteristic expansion, stock-return fit, and factor-only pricing

Factor model	Source characteristic intercept	Characteristics	Complete stock-return R^2 (%)	Factor-only relative pricing error (%)	
				Coefficient attribution	Stock-exposure attribution
3 latent + FFC6	Unrestricted	36	17.01	76.84	53.89
3 latent + FFC6	Unrestricted	153	17.70	92.08	72.90
3 latent + FFC6	Zero, jointly estimated	36	16.93	7.25	7.25
3 latent + FFC6	Zero, jointly estimated	153	17.57	3.55	3.55
5 latent	Unrestricted	36	17.06	70.93	43.16
5 latent	Unrestricted	153	17.71	127.64	94.49
5 latent	Zero, jointly estimated	36	17.01	1.42	1.42
5 latent	Zero, jointly estimated	153	17.62	0.44	0.44

This historical descriptive comparison asks how expanding the characteristic library changes stock-return fit and the pricing of the same target means. Higher complete stock-return R^2 means better fit; lower relative pricing error means a closer explanation of average target payoffs. Outcome columns are percentages; Characteristics is a count. The historical comparison uses 599 formation months, July 1964–May 2014, and the same 36 unscaled characteristic-managed targets. Complete stock-return R^2 uses the equal-month stock-loss objective and includes the source characteristic intercept when unrestricted. Factor-only relative pricing error is the sum of squared average target residuals divided by the sum of squared average target payoffs. Coefficient and stock-exposure attribution decompose the same fitted source model. Historical latent factors are full-sample fitted scores, not formation-known portfolio payoffs. Each zero-intercept model is jointly reestimated under the source characteristic-intercept restriction.

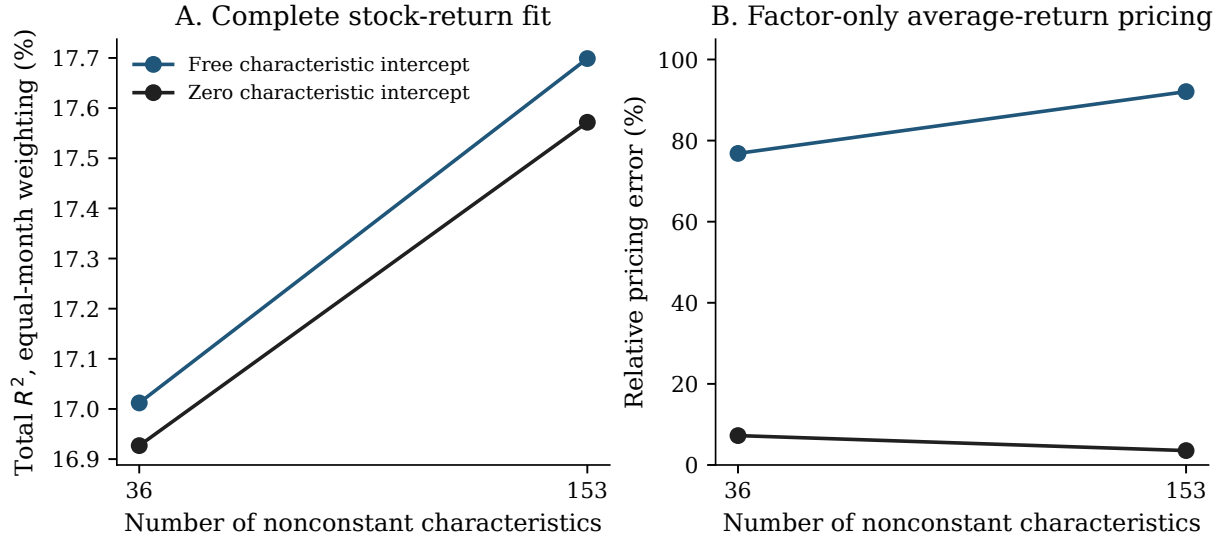


Figure 1: Characteristic expansion, stock-return fit, and average-return pricing

The historical comparison separates stock-return fit (Panel A, higher is better) from factor-only pricing of target means (Panel B, lower error is better). Both specifications contain three latent factors and FFC6, the Fama–French five factors plus momentum. The source characteristic intercept is freely estimated or jointly restricted to zero. The comparison uses 599 formation months from July 1964 to May 2014. Panel A reports complete stock-return R^2 with equal weight across months, including the source intercept when estimated. Panel B reports 100 times the sum of squared mean factor-only residuals divided by the sum of squared target means, using the same 36 unscaled characteristic-managed targets and the coefficient decomposition. Lower pricing error indicates a better explanation of these target means. The full-sample fitted factor scores are used for this descriptive comparison.

mean can be offset by a corresponding change in the characteristic intercept. The choice of decomposition is therefore relevant when one reports a factor-only premium. Zhang (2024) and Choi and Yuan (2026) examine related identification and pricing-error questions. We use this accounting exercise to identify what is being compared before turning to portfolios with directly evaluated payoffs.

A source-model characteristic intercept and a target portfolio's pricing error are different objects. The first is estimated jointly with source factor directions; the second is the portion of a target's mean payoff left outside a specified factor-compensation term. The source intercept can affect which directions the model learns without being subtracted from a subsequently traded factor payoff. The decomposition must therefore identify which component receives credit for the target's mean return.

These historical fit and pricing estimates motivate the stock-level experiment but do not evaluate a formation-known trading strategy. They use full-sample fitted factors and characteristic-weighted targets. Our subsequent tests instead form holdings from information available at portfolio formation and score their payoffs on a common return calendar. This shift takes the question from how a model divides fitted returns to what an investor could have held.

The next question is what stock exposures the richer model selects. Keeping the number of latent factors fixed forces the estimated directions to summarize a larger characteristic library. Return fit alone cannot identify which familiar stock profiles these directions represent. We therefore compare loading and characteristic profiles on the same firms, before imposing any portfolio hedge.

IV Measuring Economic Exposures

IV.A A Common Measure of Characteristic Overlap

A factor loading has an economic meaning only through the stocks to which it assigns exposure. After expanding the characteristic library, the estimated factors can fit returns better while pointing toward different firms. We therefore compare the loading profiles with familiar value, momentum, and size positions across the same stock cross section. The question is how much of each desired stock position lies in the model's loading space.

Take two characteristic combinations a and b . Applied to the current stocks, they generate profiles $Z_t a$ and $Z_t b$. Their average cross product is

$$\frac{1}{n_t} (Z_t a)' (Z_t b) = a' Q_t b, \quad Q_t = \frac{Z_t' Z_t}{n_t}. \quad (9)$$

This cross-sectional second moment, Q_t , is our exposure ruler. Its diagonal entries give mean squared characteristic values, while off-diagonal entries record mean cross products across firms. Two portfolios based on different valuation signals can have similar stock exposure even when their coefficient vectors look different. The ruler describes characteristic overlap across stocks; return covariance answers the separate question of how given stock positions share payoff risk.

Consider two valuation signals that select nearly the same firms. Their characteristic coefficients can differ, yet an investor buying and shorting the selected stocks obtains similar exposure profiles. A new signal that selects different firms can instead add a direction not already spanned by the original library. Measuring the profiles on the same stocks tells these cases apart. The return evidence then determines whether a genuinely new direction is useful for pricing or portfolio risk.

A coefficient inner product, $a'b$, changes when a characteristic is measured in different units. An investor's stock exposure need not change at all. We isolate this distinction by deterministically rescaling characteristics and inversely rescaling their coefficients, keeping every stock profile fixed. Figure IA.1 shows that empirical coverage is unchanged to numerical precision, whereas coefficient-based coverage can move. The later library experiment changes the set of available profiles; this exercise changes only their units.

Figure 2 shows how much independent stock-exposure variation is present in each library over 225 formation months. After excluding the constant and standardizing each characteristic by its second moment, the participation dimension of S_t , $(\text{tr} S_t)^2 / \text{tr}(S_t^2)$, averages 10.22 with 36 characteristics and 21.08 with 153. The five largest directions account for 58.71% and 38.82% of standardized profile variation, respectively. Thus the larger library adds distinct directions without eliminating substantial overlap. The participation dimension describes concentration across stock profiles; it is neither matrix rank nor a count of priced return factors.

The added characteristics are partly new stock exposures and partly repeated descriptions of existing ones. Projecting the 117 new characteristic profiles on the constant and original 36 leaves, on average, 51.32% of their standardized exposure variation. The residual quantifies how much of

each added profile cannot be reproduced by the baseline span. It says how much new stock-profile variation is available, not how much of it the estimated return model uses or rewards.

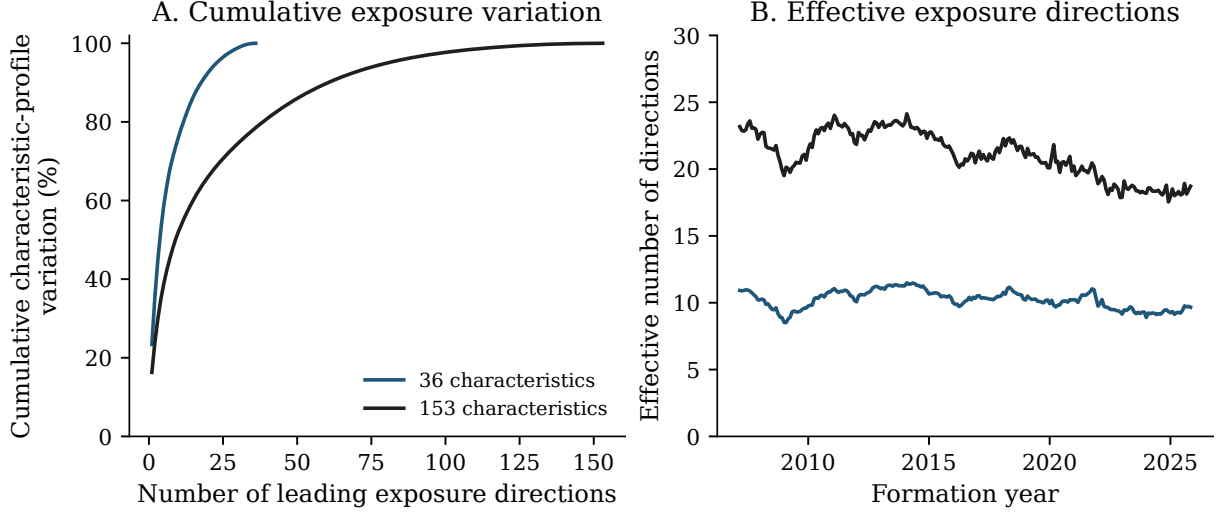


Figure 2: Characteristic-library size and distinct stock-exposure directions

The figure asks how many distinct stock-characteristic directions accompany a larger library. A steeper cumulative curve indicates greater concentration in a few directions; a larger effective number indicates a more dispersed spectrum. The figure uses 225 source-stock formation cross sections from March 2007 to November 2025. The constant is excluded, and each characteristic is standardized by its own second moment for this diagnostic. Panel A orders the eigenvalues of the resulting characteristic second-moment matrix, averages each eigenvalue's share across months, and cumulates those shares. Panel B reports the effective number of directions in each month, computed as $1/\sum_k p_k^2$, where p_k is an eigenvalue's share of the total. The blue and black lines correspond to the 36- and 153-characteristic libraries. These measures describe variation in stock characteristic profiles, not explained return variance or a count of priced factors.

To measure how much of a chosen characteristic the estimated factor loadings can reproduce, we use the squared share of its stock profile lying in the loading space. For a characteristic direction a , a loading matrix L , and reference second moment Q , define

$$\mathcal{C}(a, L; Q) = \frac{a'QL(L'QL)^{-1}L'Qa}{a'Qa}. \quad (10)$$

The numerator measures the squared size of the profile projected onto the chosen loadings, and the denominator measures the size of the original profile under the same reference Q . Their ratio is the share of characteristic exposure represented by the fitted loading space. For this diagnostic,

L denotes the selected latent or joint loading columns; the portfolio constraints below add a cash column explicitly. We compare latent B and joint $[C, B]$ spaces using the same 240-month mean formation matrix at each annual origin. Baseline coefficients are embedded in the full characteristic space with zeros for unused signals. Both libraries therefore face the same reference profiles and ruler. Coverage is a stock-profile statistic, not explained return variance.

Figure 3 traces three familiar mandates: book-to-market, 11-month momentum excluding the latest month, and larger-firm size. In the unrestricted model, expanding the library lowers average joint loading-space coverage from 55.05% to 44.77% for book-to-market, 82.27% to 62.33% for momentum, and 90.17% to 71.93% for size. These numbers describe the share of each fixed stock-characteristic profile represented by the fitted loading space; factor rotations do not alter them.

The decline extends across several economic groups. In the unrestricted models, coverage falls from 44.00% to 38.51% for value and payout, 32.87% to 23.88% for profitability, and 18.84% to 13.96% for investment. Intangibles and firm attributes provide a counterexample, rising from 35.62% to 37.59%. Across the original 36 profiles, mean coverage falls from 49.94% to 35.38%; for the added 117, it falls from 24.05% to 20.83%. A larger set of candidate characteristics need not devote more of a fixed-dimensional estimated loading space to any particular familiar position.

At the same time, the expanded model uses exposures that were unavailable in the baseline span. Under an unrestricted intercept, 38.18% of the empirical exposure energy of the 153-characteristic joint loading space lies outside the original 36 and the constant; under a zero intercept, the share is 36.52%. This statistic follows the estimated loading space as a whole, rather than averaging coverage of individual named characteristics. The model can therefore use new mixtures while allocating less of its loading space to a particular familiar tilt.

The stock-profile comparison tells us what the factor model would remove from a desired characteristic position. To build an investment portfolio, we must also impose cash neutrality and decide how much to scale the stock weights after the removal. Section V performs that conversion and evaluates subsequent returns. Table IA.1 and Figure IA.1 retain the detailed economic-group and units controls.

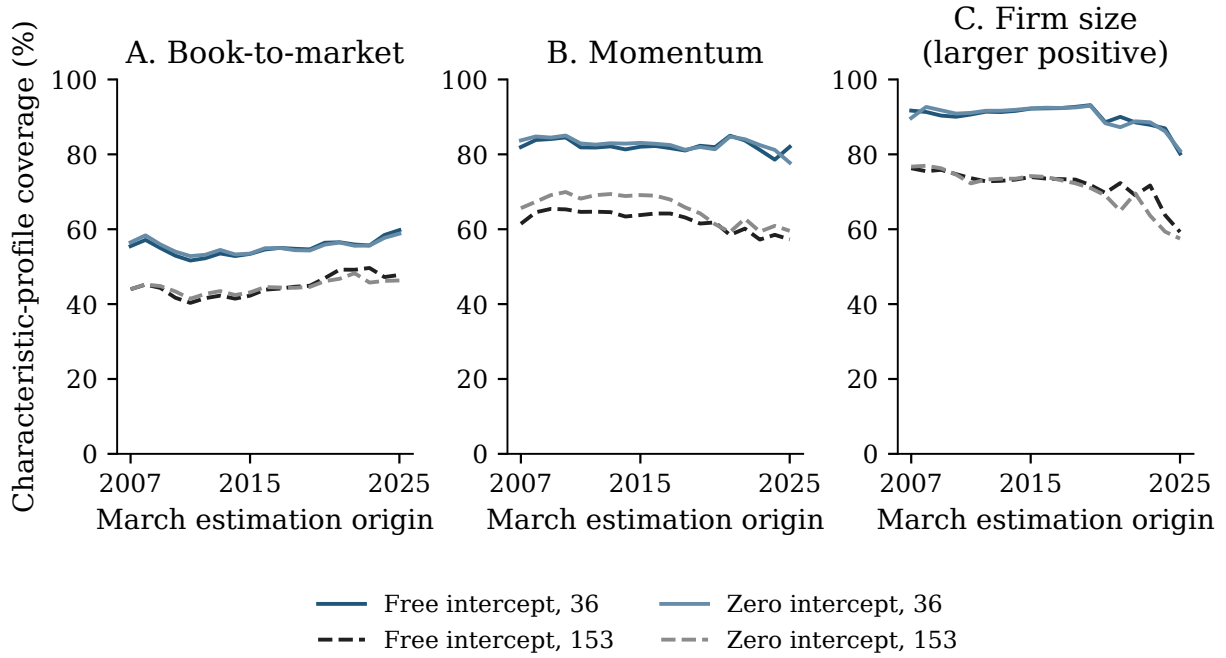


Figure 3: Coverage of familiar characteristic profiles as the library expands

Higher coverage means that more of a familiar stock-characteristic profile is represented by the estimated loading space. Each model contains FFC6 and three latent factors, estimated each March using 240 prior months. Free and zero intercept refer to the source characteristic intercept; 36 and 153 denote the numbers of nonconstant characteristics. Coverage is the squared norm of a named characteristic profile projected onto the joint observable and latent loading space, divided by the profile's squared norm. Both libraries are evaluated under the same training-average characteristic second-moment matrix at each of the 19 origins. The three reference profiles remain fixed across model specifications; individual numbered latent factors are not matched over time. Firm size is larger-firm positive. Coverage measures representation of a stock profile, not its share of realized return risk or its risk premium.

V Maintaining a Fixed Characteristic Exposure

V.A From Loading Restrictions to Stock Holdings

A factor model’s loading space becomes economically consequential when an investor uses it to restrict stock holdings. Section IV shows that this space can represent different familiar characteristic profiles as the library changes. Consider a momentum investor: some of the stocks that carry the desired tilt may also carry the estimated factor loadings. Removing those loadings can therefore change the tilt itself. We compare the resulting portfolios after restoring the investor’s original positive characteristic exposure. This holds the economic target fixed while allowing the required holdings, gross, trading, and subsequent risk to differ.

Let a select book-to-market, momentum, or firm size, and let e_0 select the constant. The annual loading matrix $L = [e_0, C, B]$ generates stock-level loading profiles $H_t = Z_t L$ at formation. We choose the stock position closest in Euclidean distance to $Z_t a / n_t$ while requiring zero net stock investment and zero exposure to every column of H_t :

$$\tilde{w}_t^Q = \frac{Z_t}{n_t} [a - L(L'Q_tL)^{-1}L'Q_t a]. \quad (11)$$

The constant column enforces zero net stock investment; the other columns remove current estimated factor loadings. Normalizing these weights by their absolute sum gives the unit-gross position w_t^N . The restriction is imposed on the stocks held at formation, rather than on their realized covariance with traded factor payoffs.

The stock-exposure ruler Q_t expresses the restriction using characteristic coordinates. Coefficient orthogonality alone does not ensure that a portfolio is neutral to the loadings of the stocks it holds. For unrestricted 153-characteristic book-to-market, coefficient projection leaves 46.10% of squared stock-weight norm in the loading-profile space on average; the current- Q projection removes that share to numerical precision. This is stock-weight overlap, not a reduction in return variance. Table IA.2 and Figure IA.3 report unit-gross and signed-tilt comparisons. We next restore the investor’s original positive characteristic exposure.

V.B Library Expansion at the Same Positive Exposure

Begin with the unhedged cash-neutral portfolio that takes the intended characteristic position. Write $s_t = Z_t a$ for its stock profile and $s_{c,t} = s_t - \bar{s}_t \mathbf{1}$ for the demeaned profile, where $\bar{s}_t$ is the cross-sectional mean. Unit-gross weights are $w_t^0 = s_{c,t} / \|s_{c,t}\|_1$, and their positive characteristic

exposure is $\xi_t^0 = (w_t^0)'s_t > 0$. The loading-neutral direction w_t^N may retain less exposure per unit gross. To restore the target, we therefore hold $w_t^N \xi_t^0 / [(w_t^N)'s_t]$. The numerator is the investor's original target; the denominator is the exposure delivered by one unit of gross in the hedged direction. Both libraries use the same source stocks, characteristic profile, and target. Gross and turnover reveal what it takes to implement that mandate; normalizing the matched holdings back to unit gross would change the mandate.

We compare loading neutrality with a historical minimum-variance hedge that preserves the same positive exposure. Within $D_a = [a, L]$, the covariance rule minimizes estimated return variance subject to current cash neutrality and ξ_t^0 (Daniel et al., 2020). Its return covariance uses complete raw characteristic payoffs in the previous 240 calendar months, with 209 to 233 observations across annual origins; estimates remain fixed until the next origin. Within each fitted model, both policies have the same attainable span but different loading exposures. Across libraries, the loading spaces are separately estimated. The experiment compares what these complete policies deliver for the same characteristic mandate.

Figure 4 translates the loading-space reallocation in Section IV into an investment comparison: the same original positive tilt can require quite different stocks and position sizes. In the unrestricted model, expansion from 36 to 153 characteristics lowers loading-neutral momentum volatility from 204.26 to 151.25 monthly basis points, mean gross from 2.34 to 1.62 times NAV, and one-way turnover from 81.40% to 57.84%. For larger-firm size, volatility falls from 275.30 to 152.12 basis points, gross from 3.27 to 1.92, and turnover from 92.99% to 51.29%. These are outcomes of separately fitted portfolio policies; Figure 4 pairs their risk with the positions required to maintain the same exposure.

The reduction in risk is not uniform across mandates. Three of the twelve library-policy contrasts have simultaneous 95% intervals excluding zero: loading-neutral larger-firm size with both intercept specifications and loading-neutral momentum with a zero intercept. Table III shows the six loading-neutral contrasts. Unrestricted momentum has an estimated change of -53.00 basis points, but the familywise interval $[-107.19, 1.19]$ includes zero; unrestricted book-to-market changes by -2.82 basis points $[-27.67, 22.02]$. Table IA.6 contains the full twelve-comparison family. All six covariance-hedge intervals include zero.

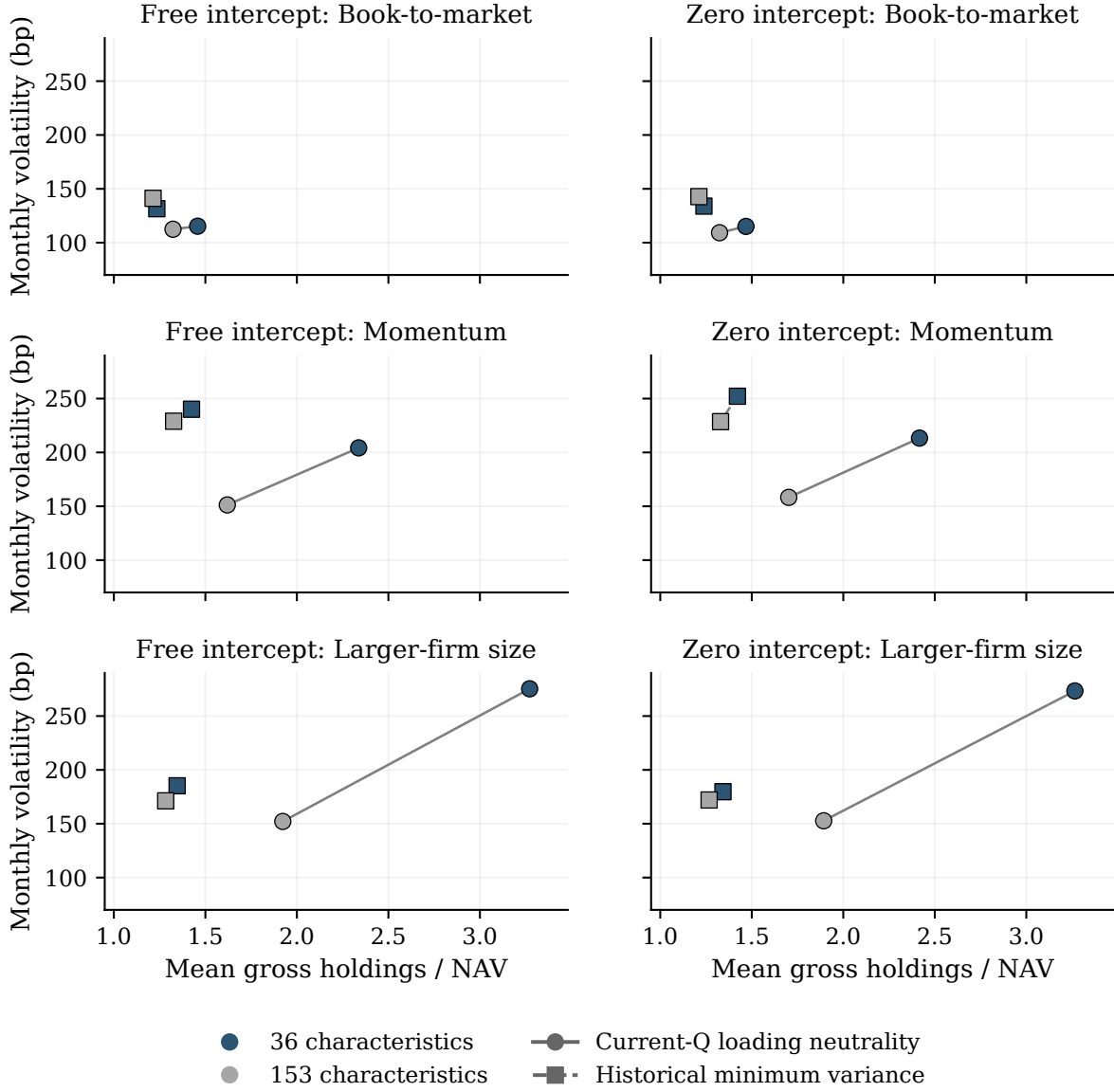


Figure 4: Portfolio risk and gross holdings under a fixed positive mandate

Each rule retains the original unhedged unit-gross portfolio's positive characteristic exposure, with zero net stock investment. Moving left indicates smaller gross holdings; moving down indicates lower subsequent volatility. Gross is not capped after matching. Colors identify libraries and markers identify risk-control objectives. Lines connect 36 and 153 characteristics within the same rule and source-intercept restriction; they are not an efficient frontier or confidence intervals. Volatility uses 216 common subsequent monthly excess returns; mean gross uses 225 formation months. Size is larger-firm positive. Table III adds mean returns, turnover, and uncertainty, with simultaneous intervals for volatility and pointwise intervals for the other metrics. Table IA.6 reports the complete twelve-comparison volatility family.

Table III: Changes in risk, gross, and trading from expanding the library at the same positive target

	Mean excess return change (bp/month) Pointwise CI	Volatility change (bp/month) Familywise CI	Mean gross change (NAV units) Pointwise CI	One-way turnover change (pp/month) Pointwise CI
<i>Free source characteristic intercept</i>				
Book-to-market	-1.29 [-10.56,9.46]	-2.82 [-27.67,22.02]	-0.135 [-0.151,-0.119]	-1.32 [-2.01,-0.69]
Momentum	16.12 [-7.18,36.28]	-53.00 [-107.19,1.19]	-0.718 [-0.764,-0.676]	-23.57 [-25.71,-21.57]
Larger-firm size	-28.52 [-62.40,7.36]	-123.18 [-166.54,-79.81]	-1.349 [-1.479,-1.223]	-41.70 [-46.02,-37.23]
<i>Zero source characteristic intercept</i>				
Book-to-market	-1.44 [-11.21,10.26]	-5.85 [-27.37,15.66]	-0.144 [-0.167,-0.124]	-3.36 [-4.30,-2.51]
Momentum	23.13 [1.57,45.36]	-55.04 [-99.43,-10.65]	-0.714 [-0.776,-0.656]	-23.79 [-25.24,-22.23]
Larger-firm size	-42.60 [-77.92,-6.49]	-120.58 [-172.92,-68.23]	-1.372 [-1.493,-1.248]	-42.39 [-47.76,-36.94]

Entries are 153-minus-36 differences for the current- Q loading-neutral rule at the original positive characteristic exposure and zero net stock investment. Negative volatility, gross, and turnover differences indicate lower subsequent risk, smaller positions, and less trading, respectively, under the larger library. The volatility column reports simultaneous 95% intervals from the twelve-comparison library family, using 19,999 shared twelve-month circular calendar-block draws and studentized max- $|t|$ inference. Mean-return, gross, and turnover brackets retain the original 1,999-draw pointwise 95% intervals. Both sets condition on fitted annual models. Returns use 216 common payoff months, gross uses 225 formation months, and turnover uses 219 valid months. Gross is absolute stock holdings per unit NAV; turnover differences are percentage points. Both libraries use FFC6 and three latent factors with separately estimated loading spaces. Size is larger-firm positive. Table IA.6 includes the six covariance-rule comparisons completing the risk family; Appendix IA.4 gives all policy levels.

Risk reduction accompanies a lower mean return for the zero-intercept larger-firm-size mandate: the 153-minus-36 excess-return difference is -42.60 basis points per month, with the original pointwise interval $[-77.92, -6.49]$. Size remains larger-firm positive, not SMB. In Table III, simultaneous inference applies to volatility; mean-return, gross, and turnover intervals are pointwise.

Risk rankings also depend on the hedge. For unrestricted 36-characteristic size, the covariance rule has 185.25 basis points of monthly volatility versus 275.30 under loading neutrality, with mean gross of 1.35 versus 3.27. For unrestricted 153-characteristic momentum, loading neutrality has lower volatility, 151.25 versus 228.99, but higher gross, 1.62 versus 1.33, and turnover, 57.84% versus 42.44%. Appendix IA.4 reports all 24 policy levels and twelve contrasts; Table IA.4 and Figure IA.4 give the within-library comparisons and pointwise uncertainty.

These hedges optimize different objects. Loading neutrality removes estimated stock-factor loadings at formation; the covariance rule minimizes historical payoff variance in the same feasible span. The neutral portfolio is feasible for the covariance problem and cannot beat its estimated historical minimum. The subsequent ranking depends on future returns. Both rules preserve the original characteristic target but can require different positions and trades.

V.C Why Restoring the Original Tilt Changes Gross Holdings

The size of a target-matched position follows from a simple ratio: the exposure the investor wants divided by the exposure that survives per unit of gross holdings. Let P_{H_t} project onto the stock-loading space H_t , including cash, and set $u_t = (I - P_{H_t})s_{c,t}$. For a nonzero residual, $\eta_t = \|u_t\|_2^2 / \|u_t\|_1$ is the exposure delivered per unit gross. Define cash-adjusted coverage $\mathcal{C}_t^{\text{cash}} = \|P_{H_t}s_{c,t}\|_2^2 / \|s_{c,t}\|_2^2$ and residual position shape $\ell_t = \|u_t\|_1 / \|s_{c,t}\|_1$. The gross needed to recover the original target along this direction satisfies

$$G_t = \frac{\xi_t^0}{\eta_t} = \frac{\ell_t}{1 - \mathcal{C}_t^{\text{cash}}}, \quad \log G_t = -\log(1 - \mathcal{C}_t^{\text{cash}}) + \log \ell_t. \quad (12)$$

Here G_t is gross holdings per dollar of NAV. The loading-overlap term increases when more squared characteristic-profile size is removed; the residual-shape term reflects the absolute stocks left after projection. Equation (12) accounts for positions along this selected direction. It neither minimizes gross across all feasible portfolios nor decomposes their subsequent return variance.

Equation (12) can be compared across libraries month by month: the 153-minus-36 change in log gross equals the change in loading overlap plus the change in residual-position shape. Averag-

ing these monthly differences preserves that exact accounting, whereas subtracting the logarithms of average gross would not. The projection uses current stocks and includes cash, unlike the training-reference coverage in Figure 3. It explains required positions; Figure 4 reports the distinct question of their subsequent volatility. Appendix IA.1.9 gives the derivation and feasibility conditions.

Figure 5 explains why the change in fitted loading-space coverage need not translate one-for-one into the change in gross holdings. With an unrestricted intercept, expansion lowers mean monthly log gross by -0.366 for momentum: reduced loading overlap contributes -0.746 , offset by 0.379 from residual-position shape. For larger-firm size, the corresponding change is -0.525 , with contributions of -1.093 and 0.567 . Thus the larger library's loading space leaves more of these familiar characteristic positions outside the hedge, but the shape of the remaining stock weights offsets part of the reduction in gross. Book-to-market has smaller changes in both components.

The accounting follows the estimated loading span, not a particular rotation of its factor basis. Although the 153-characteristic library contains the original 36 characteristics, its reestimated fixed-dimensional loading space need not contain the 36-characteristic loading space. Section VI studies a different choice: it holds annual coefficients fixed and changes the stock cross section used to measure the overlap and form the portfolio.

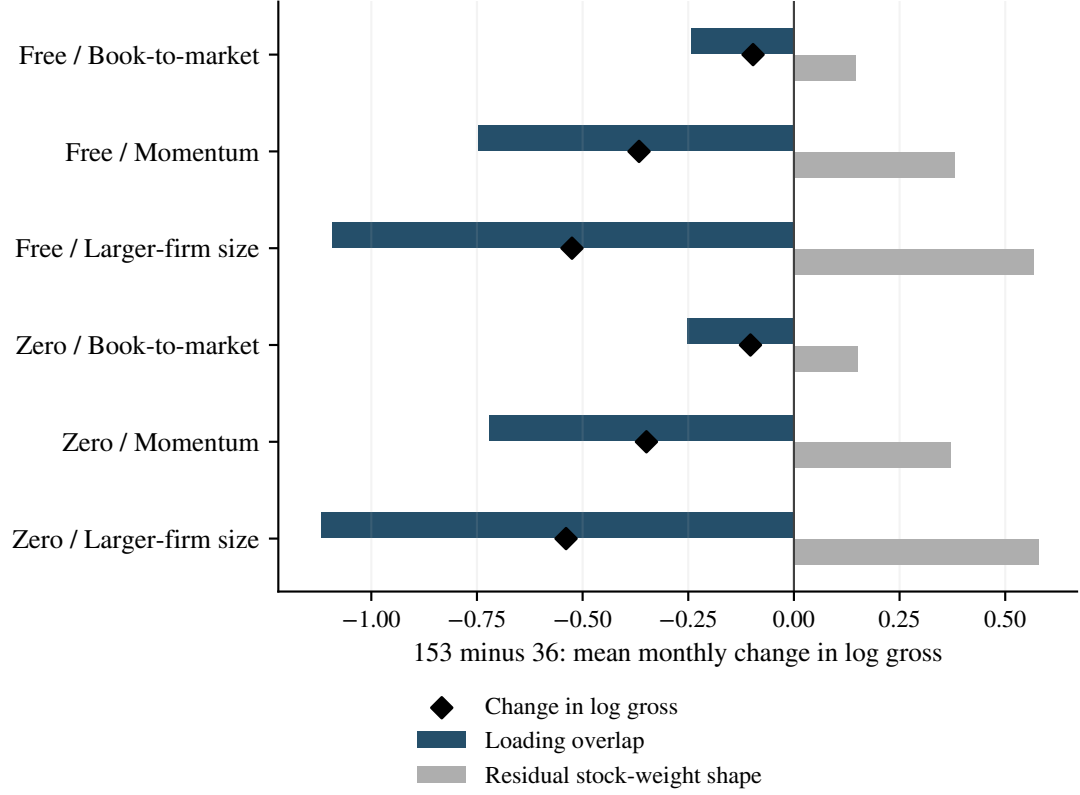


Figure 5: Why required gross holdings change when the characteristic library expands
Diamonds show the mean monthly change in log gross holdings from 36 to 153 characteristics at the same original positive target and source-intercept restriction. A negative value indicates less gross under the larger library. The bars separate this change into loading overlap and the absolute shape of the remaining stock positions. Cash-adjusted coverage uses current stocks and includes cash in the loading space. Bars average the monthly 153-minus-36 differences in the overlap term $-\log(1 - \mathcal{C}^{\text{cash}})$ and the residual-position term $\log \ell$, where ℓ is the residual-to-original ratio of absolute stock-profile sums. They sum to the mean monthly log change in gross required along the selected projection direction (diamonds). The accounting uses all 225 formation months; it does not decompose logarithms of sample-average gross or subsequent return variance. Each library uses its separately estimated loading space. Required gross is measured along the selected current-projection direction, rather than minimized over all portfolios. Figure 4 reports the separate risk outcomes.

VI Updating Exposure Information Within the Year

The loading coefficients need not change for a model’s measured stock exposures to change. Section V compares libraries that reestimate the fitted loading space. Here annual coefficients are estimated in March and held through February, while firms’ characteristics and the set of available stocks change each month. The investor faces a separate decision: should today’s loading correction be measured using the March cross section or today’s stocks? We compare the portfolios formed under these two references.

We use three exposure references. Q_{train} averages the characteristic second moments of the preceding 240 formation months; Q_{origin} uses the March cross section throughout the following year; and $Q_{\text{current}} = Q_t$ uses the stocks and characteristics available in each formation month. All policies use the same annual L , current stocks, correction span, and current cash-neutrality constraint. All form portfolios monthly. The reference changes the loading correction applied to the original characteristic profile. Only the current reference enforces neutrality to current stock loadings throughout the year; the historical references generally do not. Appendix IA.1.7 derives the projections. Current and origin positions coincide in March and first earn returns in April, although their March turnover can differ because earlier holdings differ.

VI.A Preserving the Training Rule’s Signed Exposure

Our first risk comparison matches the *signed characteristic exposure of the training-reference unit-gross portfolio*. This evaluates the position produced by an existing rule, even when it points against the investor’s original positive characteristic target. In the unrestricted 36-characteristic model, the training reference has negative momentum exposure in three months and negative larger-firm-size exposure in ten; the zero-intercept counts are one and fifteen. These months remain in the data. Table IA.30 provides the full sign record, including the absence of negative training-reference tilts in the 153-characteristic specifications.

All twelve signed-target point estimates have lower subsequent variance under the current rather than annual-origin reference. Eight simultaneous 95% intervals exclude zero: book-to-market and momentum in each model specification. All four larger-firm-size intervals include zero. For unrestricted 153-characteristic models, the estimated reductions are 32.40% [4.91,59.89] for book-to-market, 41.97% [16.58,67.35] for momentum, and 18.61% [−8.38,45.59] for size. Figure IA.5 and Table IA.13 report this separate family of twelve comparisons, with 19,999 common

calendar-block draws. The policies differ in exposure measurement and current loading restrictions, not in their monthly formation frequency.

Updating the training-average ruler to the March-origin ruler is a separate decision from updating it again within the year. For unrestricted 153-characteristic larger-firm size, the training-to-origin change reduces variance by 51.85%; the momentum estimate is -5.98% , with an original pointwise interval covering zero. These pointwise results describe replacing a long-run reference with one measured at the annual origin. Because each variance reduction has a different denominator, the sequential percentages do not add. Appendix IA.5 gives the full training-to-origin comparison and unit-gross results, where characteristic exposures are not held equal.

Risk comparisons are more informative when read alongside the stock positions needed to obtain them. Table IV reports the signed-target holdings. For unrestricted 153-characteristic momentum, monthly volatility falls from 126.99 to 96.74 basis points, but mean gross rises from 1.066 to 1.075 times NAV and turnover from 37.70% to 38.31%. The latter differences have pointwise intervals covering zero; book-to-market uses somewhat less gross and trading. All twelve mean-return differences have pointwise intervals covering zero as well. The clearer result in this family concerns subsequent variance, not a uniform gain in returns or trading.

VI.B Preserving the Original Positive Characteristic Target

We next restore the investor's *original positive characteristic target*, ξ_t^0 , rather than the signed exposure inherited from Training-Q. Each month, both Current-Q and Origin-Q portfolios match that same book-to-market, momentum, or larger-firm-size exposure. They use the same current stocks and annual loading coefficients and are cash neutral. Current-Q removes the estimated loading profiles of today's stocks; Origin-Q generally retains some of that exposure between March updates. The characteristic target is held fixed, but the comparison remains between two complete exposure-control policies with different current loading restrictions.

This twelve-comparison positive-target risk family was specified as an exploratory follow-up after the original three-reference experiment encountered nonpositive wealth in the training-reference policies. It uses the original 216 common subsequent-return months, preserves all missing wealth observations, and is assessed separately from the library-expansion and signed-target families.

Figure 6 reports the subsequent variance of those two positive-target policies. Across twelve point estimates, Current-Q has 26.62%–69.39% lower variance than Origin-Q; nine simultaneous

Table IV: Current versus annual-origin measurement: signed-target holdings and payoffs

Source intercept / no. of characteristics	Characteristic	Mean excess return change (bp/month)	Volatility change (bp/month)	Mean gross change (NAV units)	One-way turnover change (pp/month)
Free / 36	Book-to-market	-0.47	-13.70	-0.015	-0.45
Free / 36	Momentum	1.22	-32.56	0.008	0.87
Free / 36	Larger-firm size	-0.88	-11.24	-0.058	-1.25
Free / 153	Book-to-market	2.37	-17.92	-0.016	-0.39
Free / 153	Momentum	0.80	-30.25	0.009	0.61
Free / 153	Larger-firm size	1.18	-7.39	-0.010	-0.25
Zero / 36	Book-to-market	-1.42	-12.50	-0.017	-0.60
Zero / 36	Momentum	1.52	-34.72	0.002	0.56
Zero / 36	Larger-firm size	-0.38	-13.77	-0.048	-1.17
Zero / 153	Book-to-market	1.52	-18.45	-0.013	-0.32
Zero / 153	Momentum	2.29	-31.06	-0.005	0.27
Zero / 153	Larger-firm size	2.76	-9.01	-0.015	-0.37

Entries are current minus origin at the same signed training-rule characteristic exposure in each month, which may be negative. Negative volatility, gross, or turnover differences indicate a reduction in that outcome under current measurement. The rules share current stocks, annual loadings, and cash neutrality, but not the same contemporaneous loading restrictions. Mean excess returns and volatility are monthly basis points over 216 common return months; mean gross uses 225 formation months, and mean one-way turnover uses 219 eligible months. Gross differences are NAV units and turnover differences are percentage points. Each model uses FFC6 and three latent factors; Size is the larger-firm characteristic. These are differences in levels; the separate familywise variance comparison is in Table IA.13. Original pointwise intervals for all twelve mean-return differences include zero. Complete levels and pointwise intervals appear in Appendix IA.5.

familywise 95% intervals exclude zero. The nine comprise all six 36-characteristic comparisons, both 153-characteristic momentum comparisons, and zero-intercept 153-characteristic book-to-market. The other three intervals include zero: unrestricted 153-characteristic book-to-market and larger-firm size under both intercept specifications. The evidence supports lower risk for several current-reference policies at the same economic target, not a common gain across every specification.

The risk difference is only one part of the investment comparison. For unrestricted 36-characteristic momentum, mean monthly excess return moves from 25.30 basis points with Origin-Q to 18.30 with Current-Q; under a zero intercept it moves from -15.58 to -24.74 . In unrestricted 153-characteristic momentum, mean one-way turnover rises from 57.54% to 57.84% under Current-Q. Across all formation requests, the maximum gross is 12.38 times NAV for Origin-Q and 4.09 for Current-Q. Table IA.32 in Appendix IA.6 reports the complete descriptive levels, including the lower zero-intercept book-to-market mean return.

The two experiments distinguish selection of the loading space from measurement of a fixed space. A larger characteristic library changes the estimated factor-loading directions; a within-year reference choice changes how fixed annual directions are measured against current stocks. Keeping the original positive characteristic target constant makes the resulting portfolio differences economically interpretable. The signed-target exercise separately evaluates a preexisting training-reference position. We now consider other uses of factor payoffs: attribution, average-return pricing, forecasts, and traded hedges.

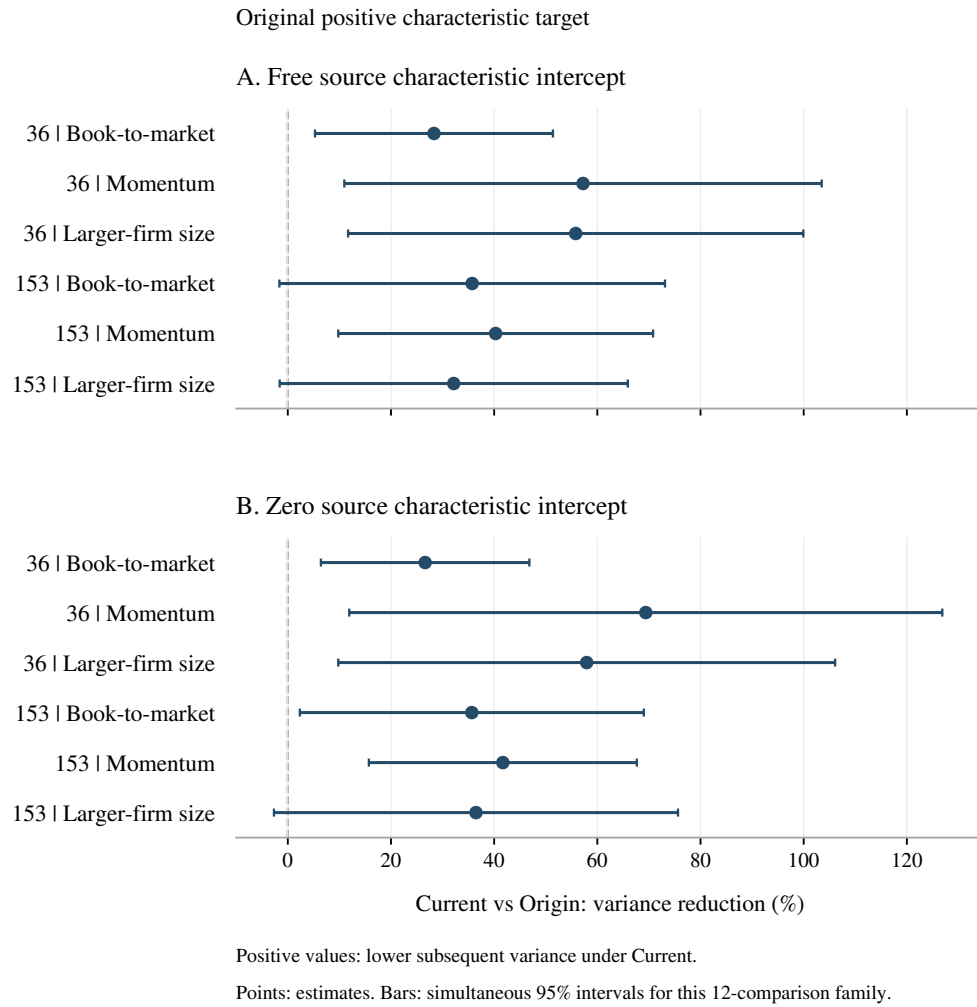


Figure 6: Current versus annual-origin variance at the same positive characteristic target. Each row compares the current-reference portfolio with the annual-origin portfolio at the same original positive characteristic exposure. Positive values indicate lower subsequent return variance under current measurement. Points are $100[1 - \text{Var}(r^{\text{current}}) / \text{Var}(r^{\text{origin}})]$ over 216 common subsequent-return months. Lines are simultaneous 95% intervals for this separate twelve-comparison family, using 19,999 common twelve-month circular calendar-block draws and studentized maximum absolute statistics. Nine intervals exclude zero; all twelve are shown. The untransformed symmetric intervals can extend above 100%; those endpoints describe approximation uncertainty, not feasible variance reductions above 100%. The family was specified after the training-reference feasibility outcome and conditions on the fitted models. Both policies form positions monthly using the same stocks, annual model coefficients, and current cash-neutrality constraint. Current measurement enforces current loading neutrality; annual-origin measurement generally does not. Free and Zero denote the source-intercept restriction; 36 and 153 count characteristics. Size is a positive larger-firm exposure.

VII Factor Compensation and Return Forecasts

VII.A Separating Observable and Latent Contributions

An estimated latent factor can carry a value, size, or momentum tilt already represented by an observable factor. The joint model may explain the target well, while the labels attached to its observable and latent contributions depend on how that shared exposure is allocated. The relevant question is therefore what is held fixed when a contribution changes. We first keep the estimated joint loading space and payoffs fixed, and change their decomposition. We then evaluate the joint factors on common target returns, where the comparison has a direct pricing interpretation.

For a fixed annual model, let $\bar{Q}$ be the mean formation matrix over its estimation window and set $A = (B' \bar{Q} B)^{-1} B' \bar{Q} C$. Define

$$C_Q = C - BA, \quad p_{t+1}^Q = p_{t+1} + Ag_{t+1}. \quad (13)$$

Then $B' \bar{Q} C_Q = 0$, while $Cg_{t+1} + Bp_{t+1} = C_Q g_{t+1} + Bp_{t+1}^Q$. The operation removes from the observable loading coefficients the component overlapping the latent loading space, and transfers the associated observable-factor payoff to the latent payoff. The joint model is unchanged. The decomposition now separates the two loading blocks using the stock-exposure distribution in the estimation window.

This transformation also gives an exact accounting for a target portfolio. When its factor slopes are transformed consistently, the observable and latent fitted contributions change by equal and opposite amounts. Replacing factor realizations by their historical means preserves the same equality for the joint factor-only forecast. The exercise therefore supplies a check on interpretation: a change in the premium attributed to one block can reflect a different allocation of shared exposure, even when the investor's joint payoff and forecast remain the same. Appendix IA.1 gives the corresponding transformations.

Figure IA.2 in the Internet Appendix applies this decomposition to the three economic test portfolios in the unrestricted 36-characteristic model. For size, the observable forecast contribution rises from 1.38 to 45.53 basis points per month, while the latent contribution falls from 0.78 to -43.37 . Their sum remains 2.16 basis points. Book-to-market and momentum exhibit the same offsetting structure. These offsetting contributions leave the joint factor-only forecast unchanged.

The joint contribution is useful when the investor wants an expected-return forecast from the available factor menu. A separate claim about an observable factor's risk premium requires ad-

ditional identification conditions. For example, Giglio and Xiu (2021) study observable-factor premia using a latent factor space that spans the relevant return variation. Our reassignment does not select a structural source of compensation. It makes the accounting convention explicit, so that differences in reported factor contributions are not mistaken for differences in the joint investment opportunity.

We use a second, return-based decomposition to isolate the latent payoff remaining after a hedge with observable factors. We regress its historical payoff on FFC6 and subtract the fitted factor slopes, leaving the regression intercept in the residual payoff. The slopes use only past observations. This operation targets covariance with observable factor returns over the calibration window. Because exposure reassignment adds only an FFC6 payoff, residualizing the native and reassigned latent payoffs produces the same residual instrument. Stock-exposure orthogonality and return-factor neutrality thus answer different questions, with an exact connection between their associated payoffs.

VII.B Average Returns on Common Test Portfolios

Table V evaluates the 36 unit-gross portfolios over the same 216 months. FFC6 produces an RMS pricing error of 20.33 basis points and relative pricing error of 61.53%. Adding latent factors from the unrestricted 36-characteristic model reduces these to 19.40 basis points and 56.02%. With 153 characteristics, the corresponding values are 18.47 basis points and 50.79%. The factor-only errors exclude the target-regression intercept, so the comparison measures the average return accounted for by the actual factor payoffs. The source stocks used to learn and construct those payoffs are separate from the target stocks.

The common targets make the pricing comparison interpretable: every model faces the same average payoffs and the same relative-error denominator. The factor component receives credit only for compensation attributed to its actual payoffs. The target-regression intercept enters the complete forecast separately. This comparison therefore asks how much of a retained portfolio's average payoff the factor menu explains.

Figure 7 shows the individual portfolios behind these aggregate errors. Each point compares the observed average payoff with the average payoff explained by the factors using slopes estimated before the evaluation returns. The 45-degree line corresponds to exact mean pricing. The vertical departure from that line is therefore economically interpretable in the same basis-point units as the portfolio's average payoff. Showing all 36 targets reveals whether a lower aggregate error

accompanies broad improvement or is driven by a small number of portfolios. Value, momentum, and size provide familiar reference points within that full comparison.

The named portfolios illustrate why an aggregate ranking is not sufficient for every mandate. In the unrestricted models, the momentum portfolio's pricing error falls from 21.74 to 15.70 basis points on expansion, while book-to-market's rises from 18.36 to 30.56. The size error changes from -3.57 to -11.65 basis points. Thus the larger model's lower overall RMS error coexists with a weaker average-return explanation for some familiar positions. The common-target design makes these differences visible: an investor can assess the model on the exposure being managed, as well as on the full set of test portfolios.

Table V: Out-of-sample factor pricing and return forecasts

Factor-payoff model	Factor-only realized-payoff fit and pricing			Predictive performance	
	Total R^2 (%)	Relative pricing error (%)	RMS pricing error (bp/month)	Factor-only R^2 (%)	Complete R^2 (%)
FFC6	45.53	61.53	20.33	0.16	0.20
Unrestricted / 36	70.25	56.02	19.40	0.02	0.50
Unrestricted / 153	67.38	50.79	18.47	0.32	0.47
Zero / 36	69.36	74.85	22.43	-0.34	0.51
Zero / 153	67.26	61.76	20.37	0.22	0.48

The table compares pricing of realized average payoffs with forecasts formed from past information. Higher R^2 and lower pricing errors indicate better performance for their respective tasks. FFC6 is the Fama–French five factors plus momentum; each other row adds three source latent portfolio payoffs and states the source characteristic-intercept restriction and library size. All models use the same 36 unit-gross target-stock portfolios and 216 common subsequent return months. Total R^2 uses realized factor payoffs and excludes the target-regression intercept; it is not a return forecast. RPE and RMS pricing error also use factor-only residuals. Predictive R^2 uses past factor means; Complete adds the separately estimated past target-regression intercept. Both forecast columns use zero excess payoff as benchmark. R^2 and RPE are percentages; RMS pricing error is monthly basis points. Source alpha is not subtracted from the traded latent payoffs.

The paired calendar-block intervals for the unrestricted 153-minus-36 difference in RMS pricing error include zero. The point estimates show that the earlier full-sample ordering need not carry over to formation-known factor payoffs and separately formed targets. The historical component statistic and the rolling payoff experiment use different estimated objects and information sets. The

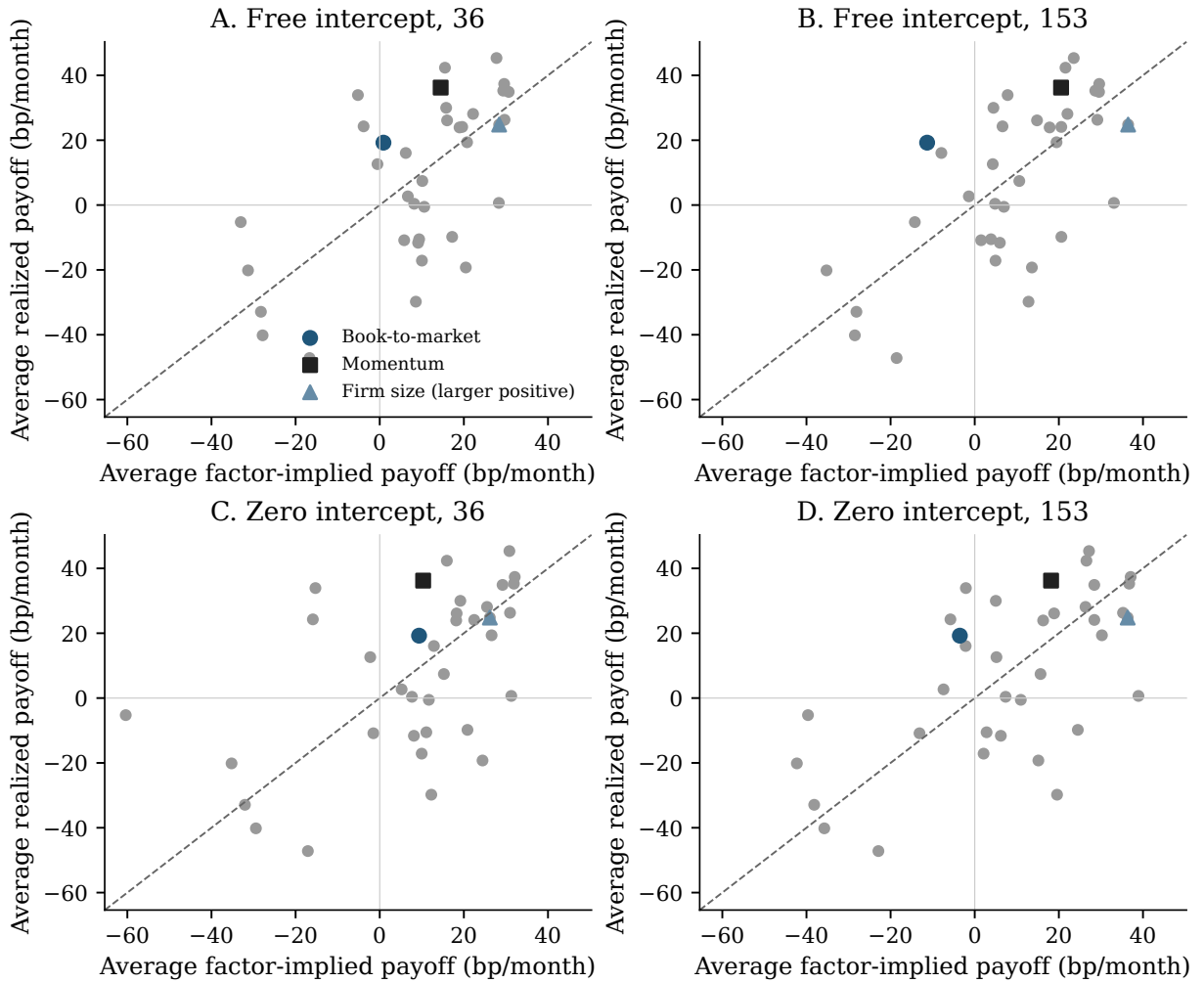


Figure 7: Average-return pricing of the same 36 characteristic portfolios

Points closer to the diagonal have a closer match between average factor-implied and realized payoffs. Each panel shows the same 36 unit-gross characteristic-managed portfolios of target stocks over 216 common return months. Source models contain FFC6 and three latent factors; free and zero intercept refer to the source characteristic-intercept restriction. The horizontal coordinate averages the realized source-factor payoffs multiplied by target slopes estimated from past returns. Source factor payoffs are constructed without subtracting the fitted source characteristic intercept, and the target-regression intercept is excluded from the displayed pricing component. Thus this is an average-return pricing comparison, not a plot of ex-ante return forecasts. The vertical coordinate is the target's realized mean payoff; the dashed line denotes exact mean pricing. Book-to-market, momentum and native larger-firm-positive size are highlighted. Grey points are the remaining targets.

appropriate conclusion concerns the observed pricing comparison, rather than a universal ordering of characteristic libraries.

VII.C Forecasting with Historical Factor Means

The forecast experiment replaces next month’s factor payoff with its historical mean. For the unrestricted models, factor-only predictive R^2 relative to zero is 0.02% with 36 characteristics and 0.32% with 153, compared with 0.16% for FFC6. Adding the estimated target intercept gives complete-forecast predictive R^2 of approximately one-half percent. These values are positive against zero, while the same complete forecasts have slightly negative R^2 against the targets’ historical means. The zero benchmark and the historical-mean benchmark describe different forecasting decisions, so we report both.

The historical-mean benchmark is demanding because it uses information from the target’s own return history. The target intercept can improve a complete forecast by absorbing a persistent average component without establishing that the component is factor compensation. The distinction follows the separate evaluation of realized-return fit, predictive fit, and pricing errors in Kelly et al. (2019, 2023b).

The evidence gives the factor model three distinct financial roles. Realized factor payoffs describe common movements in the targets. Their average compensation accounts for part of the target means. Forecasts based on historical factor means face a competitive alternative in the targets’ own historical averages. This pattern complements the stock-holdings evidence. An investor can use a factor payoff to hedge an unwanted source of variation even when its premium is difficult to forecast. The next section evaluates that use with the actual instruments available to the investor.

VIII Factor Payoffs as Hedging Instruments

VIII.A What Do Latent Factors Add to Familiar Hedges?

A latent factor is useful as a hedge only relative to the instruments already available to an investor. We evaluate that question on the separate target stocks. At each March origin, past target returns determine hedge slopes, and subsequent factor payoffs implement those hedge positions; the target-regression intercept is not traded. We compare FFC6 with three source-stock portfolios based on book-to-market, momentum, and native firm size. Those characteristic portfolios use formation-

known weights and the same source group as the latent instruments, building on the mapping from firm signals to positions in Brandt et al. (2009).

Every hedge menu is evaluated against the same target positions. The three familiar characteristic portfolios are especially useful benchmarks because an investor can construct them from the same firm information and recognize the economic exposure each one carries. The comparison therefore tests whether latent instruments add a hedge beyond a practical alternative.

Table VI shows why the choice of benchmark matters. FFC6 removes 45.66% of the sum of individual target return variances. Adding book-to-market, momentum, and size portfolios increases the reduction to 69.42%. Adding unrestricted latent portfolios instead gives 70.70% with 36 characteristics and 67.69% with 153. Latent payoffs hedge substantially more than FFC6 alone, but much of that gain is also available from three familiar characteristic portfolios.

Figure 8 asks two different questions. Panel A replaces three familiar characteristic hedges with three latent hedges, leaving nine instruments on either side; all four paired intervals include zero. Panel B adds the three latent instruments to the existing nine, producing twelve hedges. The unrestricted 36-characteristic model adds 1.81 percentage points of variance reduction, with a conditional 95% interval of 0.63 to 3.25; the 153-characteristic increment is -0.23 points. Replacement measures the value of an alternative compact menu. Addition measures incremental payoff span and changes the number of instruments.

These percentages aggregate the variances of 36 targets, rather than the return variance of a portfolio holding all targets equally. The target-by-target results in the Appendix show that hedge effectiveness differs materially across mandates. A manager concerned with one characteristic position must evaluate that exposure, even when the factor menu performs well on average.

The size target illustrates the incremental-value problem. In the unrestricted 153-characteristic model, FFC6 removes only 13.67% of the size target's variance. Adding three characteristic portfolios raises the reduction to 86.34%; adding three latent portfolios instead gives 78.33%. The native larger-firm-size position differs from the Fama–French small-minus-big factor. A strong comparison with FFC6 alone can therefore conceal the useful hedges available from characteristic-managed stock portfolios.

The investor's existing hedge menu determines the right benchmark. Replacing three familiar portfolios with three latent instruments tests whether the latent menu is a better substitute; adding latent instruments tests their value once familiar hedges are already available. We report both questions rather than treating an improvement over FFC6 as sufficient.

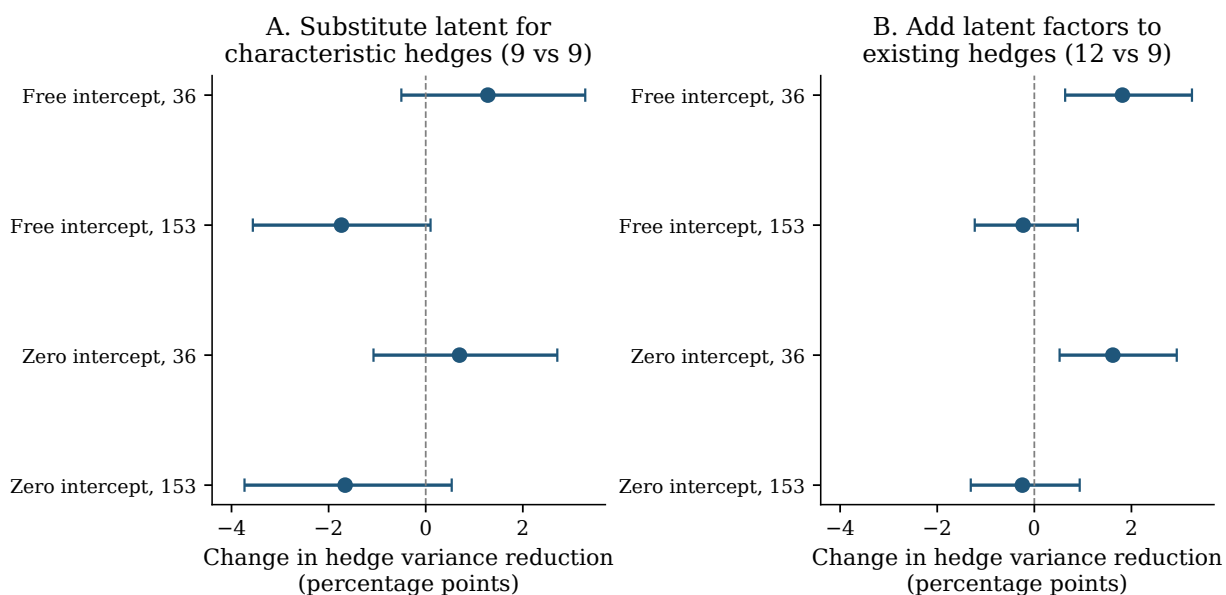


Figure 8: Latent-factor hedges beyond familiar characteristic portfolios

Positive estimates indicate an improvement over FFC6 plus three source-stock characteristic portfolios: book-to-market, momentum and native larger-firm-positive size. All comparisons hedge the same 36 unit-gross target-stock portfolios over 216 common subsequent-return months, with hedge slopes estimated from past data. Panel A replaces the three characteristic payoffs with three latent-factor payoffs, leaving nine instruments on each side. Panel B adds all three latent payoffs to the benchmark, giving twelve rather than nine instruments. The plotted difference is in percentage points of hedge variance reduction, with the sum of individual unhedged target-return variances as the common denominator. Free and zero intercept identify the source characteristic-intercept restriction. Dots and bars are point estimates and pointwise paired 95% circular calendar-block intervals (12-month blocks; 1,999 draws), conditional on the estimated source models and evaluated paths.

Table VI: Hedging with familiar and latent factor portfolios

Source intercept / no. of characteristics	Hedge variance reduction (%)			
	FFC6 only (6)	FFC6 + characteristic portfolios (9)	FFC6 + latent portfolios (9)	FFC6 + both (12)
Unrestricted / 36	45.66	69.42	70.70	71.24
Unrestricted / 153	45.66	69.42	67.69	69.19
Zero / 36	45.66	69.42	70.12	71.04
Zero / 153	45.66	69.42	67.76	69.18

The table compares familiar and latent hedge instruments for the same target portfolios. Larger entries mean a greater reduction in the sum of individual target risks. Entries are hedge variance reductions: one minus the sum of the 36 individual targets' residual time-series variances divided by the sum of their unhedged variances, expressed in percent over 216 common subsequent months. Instrument counts are in parentheses. FFC6 is FF5 plus momentum. Characteristic portfolios are source-stock book-to-market, momentum, and native firm size; size is larger-firm positive, not SMB. Latent portfolios are the three source factor payoffs. Annual target hedge slopes use completed historical returns; the target regression intercept is not a traded hedge. The first two menus are common across rows. Figure 8 reports paired uncertainty for the nine-versus-nine and twelve-versus-nine comparisons.

IX Implementing Exposure Control

IX.A How Much of the Loading Exposure Should Be Removed?

Eliminating every estimated factor loading is not the only portfolio choice. A manager may accept some loading exposure to keep more of the original characteristic tilt or avoid additional trading. We vary how much of the loading-space component is removed and measure the remaining characteristic exposure, stock positions, turnover, and subsequent risk. This gives a set of fixed policies, not an optimized hedge fraction.

Let w_t^0 be the cash-neutral unit-gross characteristic portfolio, and let u_t be its residual after projecting out the current stock-loading profiles $H_t = Z_t L$. Here u_t starts from unit-gross weights; Section V.C instead starts from the unnormalized characteristic profile. Both yield the same normalized fully neutral direction. Removing a fraction $\tau \in [0, 1]$ of the loading-space part and then normalizing gives

$$w_t(\tau) = \frac{(1 - \tau)w_t^0 + \tau u_t}{\|(1 - \tau)w_t^0 + \tau u_t\|_1}. \quad (14)$$

The numerator blends the original and fully hedged stock directions; the denominator rescales each blend to unit gross. At $\tau = 0$, the position is unhedged; at $\tau = 1$, its estimated loading exposure is removed. Intermediate policies preserve some of the original loading exposure while holding gross constant at one.

We use five common hedge fractions: 0, 25%, 50%, 75%, and 100%. Each fraction applies to every model, characteristic mandate, and formation month. No hedge fraction is fitted to the evaluation returns. These policies move from the unhedged original position to full loading neutrality; their characteristic exposure and gross respond to the chosen fraction. They are not points selected on an estimated efficient frontier.

Figure 9 shows the cost of reducing loading exposure in terms of the characteristic tilt retained. Under unrestricted 153-characteristic book-to-market, a 50% hedge keeps 94.62% of the original exposure while reducing monthly volatility from 191.98 to 143.32 basis points. A 100% hedge keeps only 75.80% of the exposure and reduces volatility to 83.09. Additional hedging lowers risk for this unit-gross policy but changes the economic position more substantially.

The same trade-off is visible for momentum and size. A 50% hedge retains 93.54% and 95.61% of their original exposures, with monthly volatilities of 205.60 and 186.43 basis points. Full hedging retains only 62.05% and 52.34%. The size profile overlaps more with the estimated loading space in this specification, so fully removing those loadings changes the larger-firm position particularly strongly. This is why unit-gross loading neutrality and preservation of the original bet are different objectives.

Table VII adds the trading requirement. Book-to-market turnover rises from 10.92% unhedged to 12.95% at 50% hedging and 18.40% at full hedging. For momentum the corresponding figures are 21.04%, 25.74%, and 35.71%. A partial hedge can thus keep more characteristic exposure with fewer trades, while accepting greater estimated loading exposure. Choosing among the policies requires an investor's own trading costs and exposure tolerances.

If the manager instead insists on the original positive tilt, each partial-hedge direction must be scaled to recover that target. Under full hedging in the unrestricted 153-characteristic model, this produces mean gross holdings of 1.32 for book-to-market, 1.62 for momentum, and 1.92 for larger-firm size. The corresponding monthly volatilities are 112.47, 151.25, and 152.12 basis points, versus 191.98, 262.05, and 231.41 for the unhedged positions. The original pointwise paired intervals for these three changes exclude zero; these contrasts are not members of the three main

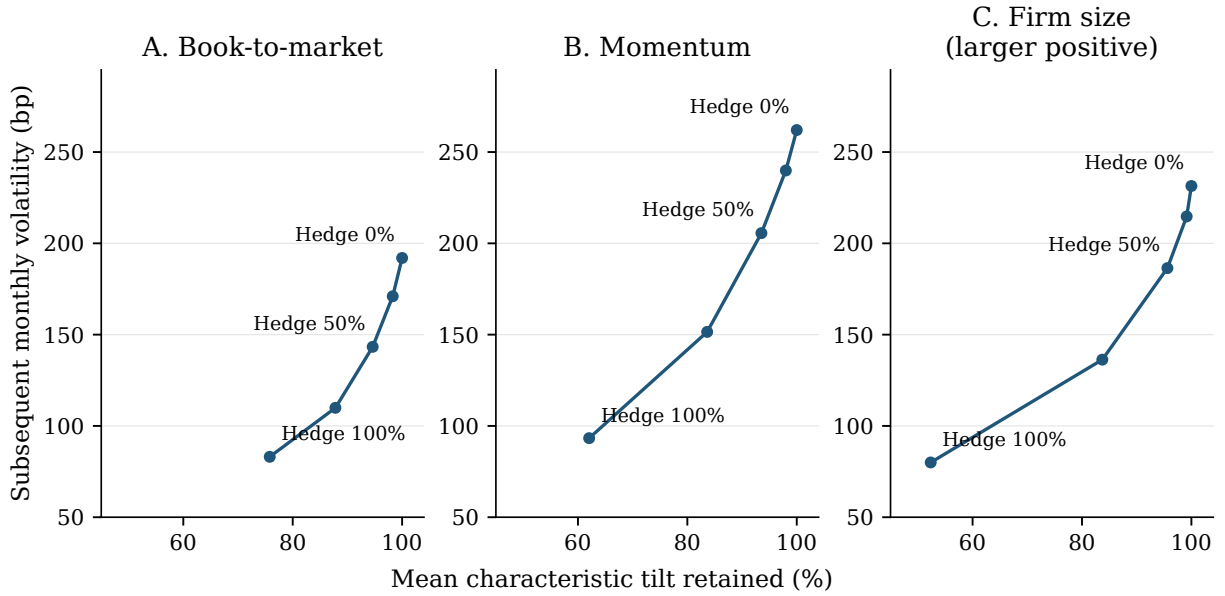


Figure 9: Volatility and retained characteristic tilt at unit gross

Moving right retains more of the original characteristic tilt; moving down lowers subsequent return volatility. The source model contains 153 nonconstant characteristics, FFC6 and three latent factors with a free characteristic intercept. The five policies remove 0%, 25%, 50%, 75% or 100% of the initial characteristic portfolio's projection onto the estimated loading profiles before normalizing stock holdings to unit gross. The labels identify that loading-hedge fraction, not a percentage reduction in risk. The horizontal coordinate averages the monthly ratio of each policy's signed characteristic tilt to the unhedged unit-gross portfolio's tilt over 225 formation months. The vertical coordinate is subsequent monthly excess-return volatility over 216 common months. Lines connect the five preset policies; they are not estimated efficient frontiers. Native size is larger-firm positive. No hedge fraction is selected using evaluation returns.

Table VII: Partial hedging, characteristic exposure, and trading

Characteristic mandate	Loading projection removed (%)	Unit gross holdings			Original positive exposure	
		Exposure retained (%)	Volatility (bp/month)	One-way turnover (% NAV/month)	Gross holdings / NAV	Volatility (bp/month)
Book-to-market	0.00	100.00	191.98	10.92	1.00	191.98
Book-to-market	50.00	94.62	143.32	12.95	1.06	152.47
Book-to-market	100.00	75.80	83.09	18.40	1.32	112.47
Momentum	0.00	100.00	262.05	21.04	1.00	262.05
Momentum	50.00	93.54	205.60	25.74	1.07	218.64
Momentum	100.00	62.05	93.30	35.71	1.62	151.25
Firm size	0.00	100.00	231.41	7.73	1.00	231.41
Firm size	50.00	95.61	186.43	12.28	1.05	195.34
Firm size	100.00	52.34	79.99	26.52	1.92	152.12

The table links the amount of loading exposure removed to the retained characteristic tilt, subsequent risk, and trading. It uses the unrestricted 153-characteristic source model. The hedge fraction is the fraction of the original loading-space projection removed before unit-gross normalization. Under unit gross, original characteristic exposure retained is the signed characteristic exposure divided by that of the unhedged portfolio, expressed in percent. The final two columns instead restore the unhedged portfolio's original positive characteristic exposure each month; they are not unit-gross portfolios. The turnover column refers only to unit-gross holdings. Formation quantities use 225 months, volatility 216 common subsequent returns, and one-way turnover 219 valid months. Volatility is monthly basis points and gross is the sum of absolute stock weights per unit NAV; turnover is percent of NAV per month. Firm size is larger-firm positive. The full five-fraction experiment also includes 25% and 75%; fractions are not selected using evaluation returns.

simultaneous-inference families. The matched-target experiment reveals the additional holdings required to retain the same bet.

The full five-fraction results for both characteristic libraries and intercept restrictions appear in the Appendix. Together, the policies show two possible costs of hedging: under unit gross, less of the original characteristic tilt may remain; at the fixed original tilt, gross holdings may have to increase. The choice depends on the investor's exposure objective and financing capacity.

IX.B When Matching the Target Requires Extreme Holdings

A fixed positive target can become expensive when a selected hedge direction retains almost none of the desired characteristic. In June 2008, the zero-intercept 36-characteristic Training-Q size portfolio delivers just 0.0000389761 of positive characteristic exposure per unit gross. Restoring

the original target of 0.335497911 then requires gross of 8,607.78 times NAV. The Origin-Q and Current-Q directions require 3.99 and 3.94 on the same stocks and target, while imposing different current loading restrictions. The extreme figure comes from scaling this particular training-reference direction, not from minimizing gross over all feasible stock positions. Appendix IA.7 traces the exposure cancellation, first nonpositive model-wealth events, and post-terminal counterfactual formations. The case shows why exposure matching and the holdings it requires must be assessed together.

X Changing Market Conditions

Both the estimated loading directions and the distribution of firms change across annual origins. Table IA.5 separates their contributions to changes in exposure coverage by averaging the two possible orders of updating them. That annual decomposition is distinct from Section VI, where annual loading coefficients stay fixed as the exposure reference changes within the year. State-based results in the Internet Appendix also indicate that hedges can remove a larger fraction of target variance in high-volatility states, without necessarily leaving lower residual variance.

We also ask whether observed exposure drift forecasts next month's squared hedge loss after conditioning on past market volatility and hedge loss. The paired intervals include zero. The practical role of a current exposure ruler is therefore its ability to impose a stock-level loading restriction at formation, rather than a demonstrated signal for timing future hedge losses.

XI Conclusion

A characteristic model becomes an investment decision when its estimated loadings are translated into stocks. Changing the characteristic library or the exposure ruler changes the positions needed to maintain the investor's chosen tilt. Expansion produces different risk and mean-return outcomes across mandates: three of twelve library comparisons have simultaneous risk intervals excluding zero. An exact gross-holdings identity shows how loading overlap and residual-position shape jointly determine the size of the selected projected portfolio.

Current stock information matters even when annual loading coefficients do not change. Relative to the annual-origin ruler, Current-Q lowers estimated subsequent variance in all twelve positive-target point estimates, with nine simultaneous intervals excluding zero. Eight of twelve signed-target comparisons are supported in their separate family. The two rules differ in contemporaneous loading restrictions; mean returns and turnover respond differently across mandates. The

result is about the risk of complete portfolios formed at a common characteristic target, not about a universal ordering of factor models.

The holdings provide the implementation lesson. If a chosen hedge direction retains almost none of the target characteristic, restoring the bet can require extreme gross positions even when other directions carry it at modest scale. Evaluating a characteristic model therefore involves four linked objects: the investor's target, the loading rule, the stocks required to implement it, and the returns those stocks subsequently earn.

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Internet Appendix

IA.1 Exposure Measurement and Portfolio Restrictions

The exposure projections require positive characteristic energy and full column rank of the stock-loading profiles on which we project. Specifically, $a'Qa > 0$, the relevant matrices $L'QL$ and $B'QB$ must be nonsingular, and the residual holdings must be nonzero before unit-gross normalization. Coefficient projections use a pseudoinverse if their combined restriction matrix has redundant columns. The empirical calculations check support and reconcile the resulting projections with direct stock-space solutions. These conditions ensure that each reported exposure and portfolio is well defined.

IA.1.1 Reassigning a Characteristic Intercept

For the source model, write the complete coefficient vector as $h_{t+1} = \Gamma_\alpha + Cg_{t+1} + Bf_{t+1}$. Let Q be a positive-definite reference characteristic second moment on the active coefficient space, and assume B has full column rank. Define $d_Q = (B'QB)^{-1}B'Q\Gamma_\alpha$, $\Gamma_{\alpha,Q} = \Gamma_\alpha - Bd_Q$, and $f_{Q,t+1} = f_{t+1} + d_Q$. These definitions separate the part of the characteristic intercept lying in the latent loading space from its exposure-orthogonal remainder.

Proof. First, $B'Q\Gamma_{\alpha,Q} = B'Q\Gamma_\alpha - B'QB(B'QB)^{-1}B'Q\Gamma_\alpha = 0$. Second, $\Gamma_{\alpha,Q} + Bf_{Q,t+1} = \Gamma_\alpha - Bd_Q + Bf_{t+1} + Bd_Q = \Gamma_\alpha + Bf_{t+1}$. Adding Cg_{t+1} and multiplying by Z_t therefore preserves the complete fitted stock return in every month. Averaging this identity over any common set of stocks and dates preserves the complete fitted mean. Omitting $\Gamma_{\alpha,Q}$ after the transformation removes a different component than omitting Γ_α before it. $\square$

The identity explains why a reported factor-only pricing error can change even when the complete model fit does not. A new zero-intercept estimation is different: it optimizes the factor directions subject to the restriction, rather than reassigning a component of an existing fit.

IA.1.2 Observable and Latent Factor Contributions

For $A = (B'QB)^{-1}B'QC$, define $C_Q = C - BA$ and $p^Q = p + Ag$. Then $B'QC_Q = 0$ and $Cg + Bp = C_Qg + Bp^Q$. If a target payoff uses slopes (b_g, b_p) , its coefficient transport is $b_{g,Q} = b_g - A'b_p$ and $b_{p,Q} = b_p$. Substitution gives

$$b'_{g,Q}g + b'_{p,Q}p^Q = (b_g - A'b_p)'g + b'_p(p + Ag) = b'_g g + b'_p p. \quad (\text{IA.1.1})$$

Historical means preserve the same identity when they use the same observations. Thus both the realized joint fit and its forecast are invariant, while the two reported contributions can change.

The reference matrix Q does not impose a restriction on $\text{Cov}(g, p^Q)$. That covariance equals $\text{Cov}(g, p) + \text{Var}(g)A'$, which need not vanish when $B'QC_Q = 0$. To remove observable return-factor exposure over a calibration sample, regress p on a constant and g and retain the slope matrix D . The residual instrument is $p - D'g$. Regressing $p + Ag$ yields slopes $D + A'$, leaving the identical residual instrument. The intercept remains in that instrument's mean. This shows precisely how stock-exposure attribution and historical return-factor neutralization differ.

IA.1.3 Exposure Coverage and Units

The projection of the stock profile Za onto ZL is $ZL(L'Z'ZL)^{-1}L'Z'Za$. Its squared norm divided by $\|Za\|^2$ simplifies to (10). This statistic lies between zero and one because it is the squared norm of an orthogonal projection divided by the original squared norm. It measures how much of a named characteristic profile the fitted loading space can represent, independently of an arbitrary rotation of the columns of L .

Let D be any nonsingular diagonal change of characteristic units. Set $Z^* = ZD$, $Q^* = D'QD$, $a^* = D^{-1}a$, and $L^* = D^{-1}L$. Then $Z^*a^* = Za$ and $Z^*L^* = ZL$, so the stock-space projection and its coverage are unchanged. Equivalently, substitution cancels D from every term in (10). This is a units-invariance statement for a fixed exposure measure. A different stock population or stock weighting can change that measure and therefore the economic question.

IA.1.4 Holdings with Loading Restrictions

Consider $\min_w \frac{1}{2} \|w - v\|^2$ subject to $H'w = 0$, where $H = ZL$ has full column rank. The Lagrangian is $\frac{1}{2} \|w - v\|^2 + \eta'H'w$. Here η is the vector of Lagrange multipliers for the loading restrictions, local to this proof. Its first-order condition gives $w = v - H\eta$. Imposing the constraint yields $\eta = (H'H)^{-1}H'v$. For $v = Za/n$, with n the number of stocks in this cross section, substitution gives (11). The Hessian is the identity, so this feasible solution is the unique minimizer. Unit-gross normalization preserves all homogeneous loading restrictions.

The coefficient-distance comparison solves a different projection with the same cash restriction. Write $m = Z'\mathbf{1}/n = Qe_0$ and $K = [L, m]$. Its coefficient vector is $a_E = a - K(K'K)^\dagger K'a$, where the dagger denotes the Moore–Penrose inverse. This minimizes $\|b - a\|^2$ subject to $L'b = 0$ and $m'b = 0$. Holdings Za_E/n therefore have zero net investment while retaining coefficient orthogonality. The stock-loading restriction would instead require $L'Qb = 0$. The stock-exposure holdings

are checked against direct stock-space projection, and the coefficient rule is checked against an independent singular-value projection.

Let w^E and w^Q be the unit-gross portfolios. Their desired characteristic exposures are $(w^E)'Za$ and $(w^Q)'Za$. The matched-exposure control scales w^Q by their ratio, using formation information only. This preserves the loading restrictions and matches the signed characteristic exposure, while allowing gross holdings to differ from one. A negative ratio matches an oppositely signed characteristic exposure; such cases are retained and counted. Matching exposure therefore does not impose the same leverage budget. The control quantifies the budget required to pursue both goals.

IA.1.5 Pricing Restrictions

The zero-intercept models impose $\Gamma_\alpha = 0$. An empirical bound such as $\Gamma'_\alpha Q \Gamma_\alpha \leq \delta$ instead limits the squared size of the characteristic intercept across stocks. For positive-definite Q and $\delta = 0$, it implies $\Gamma_\alpha = 0$; for positive δ , it allows a nonzero intercept. Such a norm bound by itself does not establish an arbitrage restriction. The empirical comparisons in this paper use unrestricted and zero-intercept IPCA and explicitly specified portfolio constraints, rather than treating a change of ruler as proof of no arbitrage.

IA.1.6 Partial Loading Hedges

Let $P_t = H_t(H_t'H_t)^{-1}H_t'$ be the orthogonal projection onto the full-rank loading profiles, including the constant, and let the initial w_t^0 be cash-neutral with unit gross holdings. Write $u_t = (I - P_t)w_t^0$ and $v_t = P_t w_t^0$. Since P_t is an orthogonal projection, $u_t'v_t = 0$. The unnormalized partial hedge is $u_t + (1 - \tau)v_t$. Its loading-space projection is exactly $(1 - \tau)v_t$, so the projected share of its squared stock-weight norm is

$$\frac{(1 - \tau)^2 \|v_t\|^2}{\|u_t\|^2 + (1 - \tau)^2 \|v_t\|^2}. \quad (\text{IA.1.2})$$

Unit-gross normalization leaves this ratio unchanged. It also preserves cash neutrality because both u_t and $v_t = w_t^0 - u_t$ have zero net stock investment. This identity describes the imposed loading reduction; the variance of the realized payoff is computed from subsequent stock returns.

For each τ , desired exposure is $w_t(\tau)'Z_t a$. The exposure-matched comparison multiplies the partial hedge by $s_t(\tau) = (w_t^0)'Z_t a / [w_t(\tau)'Z_t a]$. Here $s_t(\tau)$ is a scalar holdings multiplier, distinct from the stock profile s_t used in the gross decomposition. All terms use formation information. The denominator is checked to be positive in this experiment, and every model, anchor, month, and

fixed hedge fraction is retained. Scaling changes gross positions and realized risk while keeping the characteristic exposure equal to that of the initial portfolio.

The implementation reconstructs u_t from the unit-gross fully projected portfolio w_t^Q . Since $u_t = c_t w_t^Q$ and $u_t'(w_t^0 - u_t) = 0$, the scale is $c_t = (w_t^0)' w_t^Q / [(w_t^Q)' w_t^Q]$. We verify this orthogonality and recover the original unhedged and fully projected holdings at $\tau = 0$ and $\tau = 1$ for every month. New partial portfolios are then evaluated directly on the stored stock returns; their turnover uses their own return-adjusted preceding holdings, rather than an average of endpoint turnover statistics.

The partial-hedging confidence intervals compare subsequent monthly volatility with the original unhedged portfolio on the same return calendar. Each block draw applies jointly to both rules and retains the common missing-month mask. We report both unit-gross and equal-characteristic-exposure comparisons. The intervals condition on the fitted models and the fixed hedge fractions; they do not estimate an investor's optimal fraction or a trading-cost parameter.

IA.1.7 Current, Training, and Return-Covariance Measures

Let $m_t = Z_t' \mathbf{1} / n_t = Q_t e_0$. A residual coefficient vector $a - L\theta$ generates holdings $Z_t(a - L\theta) / n_t$, whose net stock investment is $m_t'(a - L\theta)$. For a reference matrix M , consider

$$\min_{\theta} \frac{1}{2} (a - L\theta)' M (a - L\theta) \quad \text{subject to} \quad m_t'(a - L\theta) = 0. \quad (\text{IA.1.3})$$

The frozen-training rule sets $M = \bar{Q}$, the average of the same 240 formation matrices used by the annual model. The comparison based on historical returns sets $M = \hat{\Sigma}_x$. Both use the same correction span and current cash constraint. The current-ruler rule sets $M = Q_t$. The annual-origin rule sets $M = Q_{\text{origin}}$, measured on source stocks at March month-end and held through February. In every case m_t uses current stocks, so the cash constraint is current even when the reference is frozen.

Write $G = L'ML$, $v = L'm_t$, $d = m_t'a$, and $\theta_0 = G^{-1}L'Ma$. In this solve, G is the Gram matrix of the correction directions, and θ_0 gives their unconstrained projection coefficients. The cash constraint adjusts those coefficients using v and the required cash exposure d . If G is positive definite and $v'G^{-1}v > 0$, the solution is

$$\theta_M = \theta_0 + G^{-1}v \frac{d - v'\theta_0}{v'G^{-1}v}. \quad (\text{IA.1.4})$$

To verify it, the first-order conditions are $G\theta - L'Ma + v\nu = 0$ and $v'\theta = d$. Substitution yields $\nu = (v'\theta_0 - d)/(v'G^{-1}v)$ and (IA.1.4). Positive definiteness makes the constrained solution unique. When $M = Q_t$ and e_0 is a column of L , the unconstrained residual already has zero current cash exposure, so the adjustment term vanishes. Dividing holdings by their gross sum preserves cash neutrality.

For the return-covariance comparisons, $\widehat{\Sigma}_x$ is the centered sample covariance, with denominator $T_{\text{cal}} - 1$, of the full vector of raw source-stock characteristic-managed excess payoffs. The calibration set retains complete reported payoffs from the annual model's preceding 240 calendar months, with return dates no later than the origin. There are 209 to 233 observations. We apply no covariance shrinkage or penalty. Only projected covariance matrices enter the solves; no inverse of the full characteristic-payoff covariance is required.

IA.1.8 A Common Positive Characteristic Exposure

Let w_t^0 be the demeaned characteristic-managed portfolio normalized to unit gross holdings, and let $\xi_t^0 = (w_t^0)'Z_t a > 0$ be its signed characteristic exposure. With $D_a = [a, L]$, portfolios in the common attainable span have holdings $w_t = Z_t D_a \eta / n_t$. Set

$$V = D_a' \widehat{\Sigma}_x D_a, \quad A_t = [D_a' m_t, D_a' Q_t a], \quad h_t = (0, \xi_t^0)'. \quad (\text{IA.1.5})$$

The exposure-constrained covariance portfolio minimizes $\eta' V \eta / 2$ subject to $A_t' \eta = h_t$. Here η is a vector of portfolio coefficients in the attainable span, whereas the scalar η_t in the gross identity measures exposure per unit gross. The matrix V evaluates historical payoff variance, and the two columns of A_t map coefficients into cash and characteristic exposure. The constraints impose current cash neutrality and the desired positive characteristic exposure. Assuming V is positive definite and A_t has full column rank, its solution is

$$\eta^* = V^{-1} A_t (A_t' V^{-1} A_t)^{-1} h_t. \quad (\text{IA.1.6})$$

The first-order condition $V\eta + A_t \mu = 0$ and the constraints give this expression. Strict convexity establishes the minimum. We do not renormalize these holdings afterward.

The current-ruler portfolio is rescaled to the same ξ_t^0 . It belongs to the same span, satisfies both constraints, and is consequently a feasible comparison in the historical variance problem. In every model, characteristic, and month, the covariance solution has no larger estimated his-

torical variance. Its subsequent risk can differ because the covariance was estimated from past payoffs. Independent checks solve the constrained least-squares problem directly on centered historical payoff observations using a singular-value null-space basis, rather than using covariance normal equations. They reproduce the portfolio coefficients and verify feasibility and first-order conditions.

The calculations retain all four annual source specifications, three fixed characteristics, and 225 formation months. All projected metric matrices are full rank; no pseudoinverse or regularization is used in these new rules. Cash and characteristic constraints are checked before scaling and against actual stock holdings. The current-rule holdings reproduce the preceding experiment exactly. Payoff comparisons use 216 common return months, and turnover uses the same 219 months with observed prior held returns. Paired uncertainty uses the existing 12-month calendar-block protocol, conditional on source estimates and evaluated paths. The replication design log records the sequence from frozen-ruler comparison to covariance and fixed-positive-exposure controls.

IA.1.9 Gross Holdings for the Projection-Based Positive Mandate

Use the stock profile $s_t = Z_t a$, its centered version $s_{c,t}$, and $H_t = Z_t[e_0, C, B]$ from Section V. Suppose $s_{c,t} \neq 0$ and $u_t = (I - P_{H_t})s_{c,t} \neq 0$. The orthogonal projector is uniquely defined by the column space; in the full-rank case it is $H_t(H_t' H_t)^{-1} H_t'$. Because the cash column is in this space, both the projected residual and the associated portfolio have zero net stock investment.

Orthogonality gives $u_t' s_{c,t} = u_t' u_t$, and the cash restriction gives $u_t' \mathbf{1} = 0$. Therefore the unhedged unit-gross portfolio has tilt $\xi_t^0 = \|s_{c,t}\|_2^2 / \|s_{c,t}\|_1$, while the unit-gross residual has tilt $\eta_t = \|u_t\|_2^2 / \|u_t\|_1$. Scaling this residual by ξ_t^0 / η_t restores the original positive tilt. Since its absolute weights sum to one, that scaling is exactly its resulting gross holdings.

Pythagoras gives $\|u_t\|_2^2 = (1 - \mathcal{C}_t^{\text{cash}}) \|s_{c,t}\|_2^2$. Substituting the two tilts yields $G_t = \ell_t / (1 - \mathcal{C}_t^{\text{cash}})$ and then (12). Subtracting this log equality across two fitted libraries proves the paired monthly decomposition. Averaging those monthly equalities preserves exact accounting. It does not equate the mean log difference to the difference between logarithms of mean gross holdings.

The identity describes holdings along a specified projected residual, rather than a minimum-gross optimization over all portfolios satisfying the constraints. It imposes neither $G_t \geq 1$ nor a future-return risk ordering without further conditions. A zero residual makes the original positive target infeasible along this direction, and an unvarying original profile makes its positive mandate

undefined. Such cases must be identified before forming the portfolio, rather than repaired by truncating a denominator. Subsequent return variance remains a separate empirical outcome.

The empirical accounting is evaluated for all four source-model specifications, three predetermined characteristics, and 225 formation months. Current stock profiles and the cash-inclusive loading matrix are reconstructed with the formation transformations used for the portfolio rules. Direct stock-space projections and characteristic-space Q_t projections give the same residual holdings, which reproduce the archived positive-target portfolios. Monthly log terms are computed before taking the library difference and time average. All 2,700 model–characteristic–month records have nonzero original and residual profiles and full-rank projected loading spaces; no denominator truncation or infeasible-month deletion is used.

IA.2 Data Construction and Estimation

The raw inputs comprise annual U.S. JKP characteristic files, CRSP CIZ monthly return and trading-status extracts, and monthly observable factors and the risk-free return. We preserve the exact vintages used for the experiment. The code checks unique security-month keys, duplicate CRSP identifiers within a formation cross section, finite processed characteristics, and the formation/return calendar. The private replication archive includes those inputs, their checksums, the preprocessing code, intermediate moments, and saved estimates.

We rank characteristics before imposing the positive-capitalization filter or the source/target assignment. Missing raw values receive the centered rank zero, representing a neutral characteristic signal under this transformation. The source/target assignment uses the first eight bytes of the SHA256 hash of the fixed seed and integer CRSP identifier, modulo two. The seed and exact rule are recorded in the replication code. The dictionary, rank transformation, and assignment stay fixed across all compared models.

Stock-loss moments use formation stocks with observed subsequent excess returns: $W_t^o = (Z_t^o)'Z_t^o/n_t^o$, $x_{t+1}^o = (Z_t^o)'r_{t+1}^o/n_t^o$, and $s_{t+1}^o = (r_{t+1}^o)'r_{t+1}^o/n_t^o$. Formation exposure moments Q_t use all eligible formation stocks, before subsequent-return availability is known. For any coefficient vector h , the stock mean squared error equals $s_{t+1}^o - 2h'x_{t+1}^o + h'W_t^oh$. This identity permits low-memory estimation while keeping the stock-return objective distinct from the exposure population.

The source-stock holdings underlying the first term of (4) are rows of $(B'Q_tB)^{-1}B'Z_t'/n_t$. The observable-factor positions are rows of $-(B'Q_tB)^{-1}B'Q_tC$. These positions use formation characteristics and previously estimated coefficients. A missing return on a held source stock makes

the source index unavailable for that month; the code does not renormalize the surviving holdings. These gross factor indices are used in target pricing and payoff hedges. The separate stock-holdings experiment constructs all of its positions directly and does not infer stock-level trading costs for the external FFC6 factors.

Each March model uses 240 prior monthly stock-loss moments. Formation dates in those estimation windows precede the model origin and return dates do not exceed it. Target calibration uses only completed returns available at the origin. Monthly factor-mean and target-mean forecasts use a matched set of complete historical observations. We retain all 225 evaluation formation months in stored paths, marking unscored return months rather than collapsing the calendar. This also preserves the spacing of observations used in block inference and horizon-based forecasts.

For the incremental-exposure calculation, partition the formation matrix into the baseline indices I (constant and 36 characteristics) and additions A . The residual second moment is $Q_{AA} - Q_{AI}Q_{II}^{-1}Q_{IA}$. We divide its diagonal by the original additions' second moments and average across the 117 additions. The statistic is the remaining standardized exposure energy after projection on the baseline. It does not estimate mutual information or determine whether an addition earns a risk premium.

To measure how much of a joint loading space lies outside the baseline characteristic span, first normalize its columns so that $\tilde{L}'Q\tilde{L} = I_k$. If P_I is the Q -orthogonal coefficient projection onto the baseline, the share is $k^{-1} \text{tr}[\tilde{L}'Q(I - P_I)\tilde{L}]$. This average across normalized loading directions is invariant to their rotation. It differs from averaging the coverage of each named characteristic: one statistic measures the selected space, while the other measures individual economic reference profiles.

IA.2.1 A Fixed-Mean Projection Diagnostic

For the historical comparison, let $\bar{y}$ be the fixed 36-target mean and let $\bar{M}$ map constant joint factor prices to those targets using average target loadings. The statistic $1 - \min_{\lambda} \|\bar{y} - \bar{M}\lambda\|^2 / \|\bar{y}\|^2$ measures how well the selected factor space can span the target mean when prices are chosen freely on that mean. It excludes the source characteristic intercept. In the unrestricted five-latent-factor specification it falls from 58.72% to 40.06% on expansion; under the zero-intercept restriction it rises from 95.67% to 96.99%. An invertible change of factor coordinates leaves this diagnostic unchanged. It is distinct from both monthly cross-section R^2 and a feasible forecast.

IA.3 Additional Exposure and Implementation Comparisons

The additional comparisons examine changes in characteristic units, economic-group coverage, factor attribution, and the translation of loading restrictions into stock holdings. The two-reference update plot distinguishes the training rule's signed tilt from the positive original tilt in the covariance comparison. It complements the exposure-reference comparisons by showing the corresponding endpoint and alternative risk-control objective. These earlier displays retain their stated pointwise intervals; the separate familywise tables below provide inference for the central portfolio comparisons.

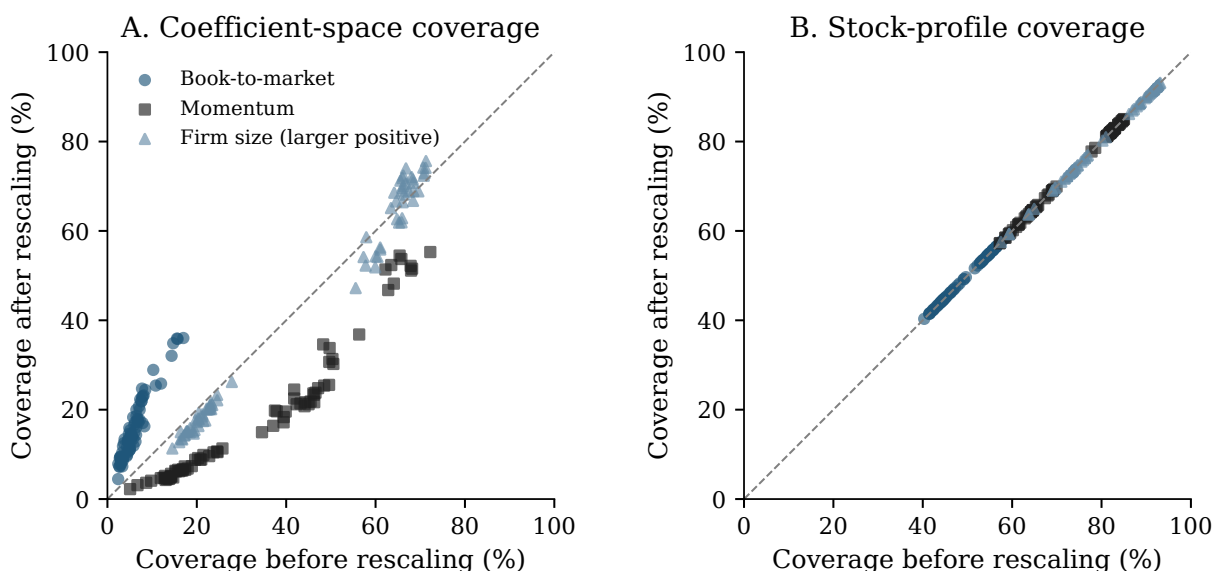


Figure IA.1: Changing characteristic units while preserving every stock exposure

Points on the diagonal preserve measured coverage when characteristic units change. Each point is a fixed economic characteristic, source-model specification and annual estimation origin. The figure pools three characteristics, four source specifications and 19 March origins from 2007 to 2025. Each characteristic is multiplied by a deterministic scale between one-half and two; loading coefficients and the reference characteristic direction are transformed inversely, preserving their stock profiles without reestimation. The horizontal and vertical axes show coverage before and after this rescaling. Panel A measures projection in coefficient space; Panel B measures projection of the same stock profiles under the empirical characteristic second moment. The dashed line denotes unchanged coverage. Firm size has its native larger-firm-positive orientation.

Table IA.1: Joint exposure coverage across economic groups

Economic group	Characteristics in full library	Joint loading-space coverage (%)			
		Free source intercept		Zero source intercept	
		36	153	36	153
Accounting quality	17	13.38	10.30	13.41	10.58
Composite signals	7	31.53	22.74	33.32	25.37
Financing and leverage	15	27.19	22.20	28.39	23.60
Intangibles and firm attributes	9	35.62	37.59	35.88	37.58
Investment	23	18.84	13.96	18.64	13.56
Trading liquidity	9	57.36	45.18	57.17	43.36
Momentum	10	48.80	36.14	48.66	37.89
Profitability	22	32.87	23.88	33.49	25.43
Historical return risk	18	33.75	27.62	34.34	28.72
Seasonality	10	9.51	9.26	9.49	9.35
Value and payout	13	44.00	38.51	44.41	39.09

Higher entries mean that the estimated loading space represents more of the stock-characteristic profiles in that economic group. All four models contain FFC6 and three latent factors. The same 153 reference characteristic profiles are evaluated under every model; baseline loadings are embedded with zero coefficients on unused characteristics. Count refers to the full 153-characteristic library. Joint loading-space coverage is the fraction of each characteristic profile's squared norm represented by the observable and latent loading profiles under a common past-240-month formation Q. We average within each fixed economic group and then over 19 annual estimation origins. Entries are percentages. Unrestricted and Zero refer to the source characteristic intercept, not a target-regression intercept. These descriptive group averages are not shares of return risk.

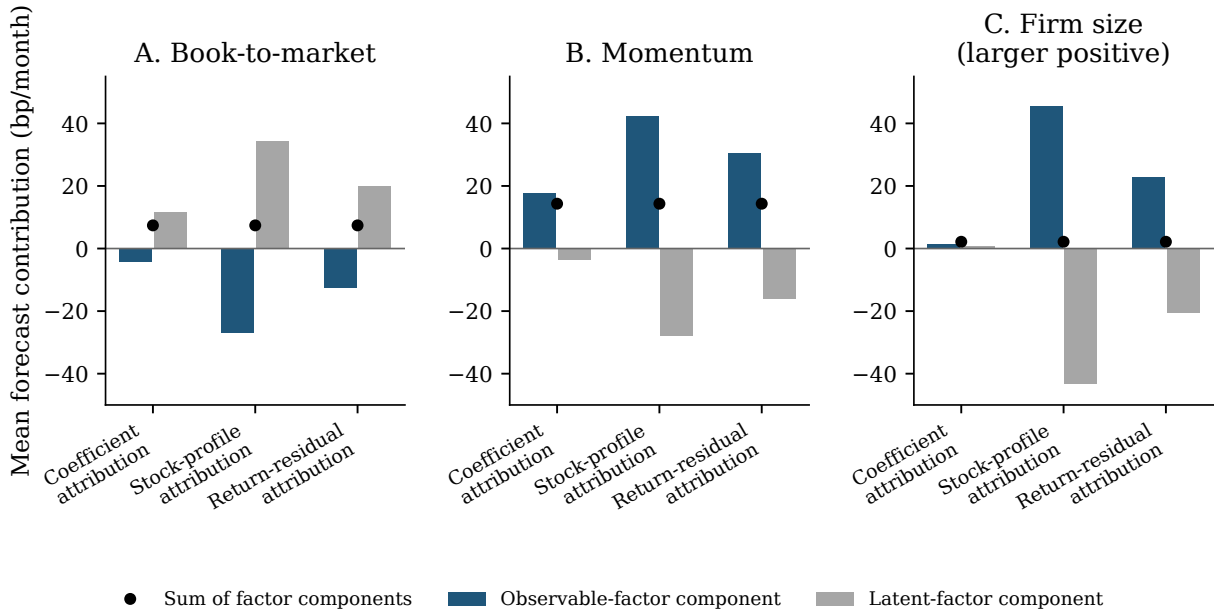


Figure IA.2: Different factor attributions of the same return forecast

Bars show how the same forecast is allocated between observable and latent factors; the black dot is their unchanged sum. The figure uses the 36-characteristic source model with a free characteristic intercept and shows averages over 216 common evaluation months. Each category decomposes the same factor-only forecast for a fixed target portfolio.

Forecast factor means use past observations. Blue and grey identify observable- and latent-factor components, respectively; the target-regression intercept is excluded. Coefficient attribution uses the original fitted decomposition.

Stock-profile attribution reallocates overlap using the empirical characteristic second moment. Return-residual attribution reallocates historical FFC6 return exposure from the latent component to the observable component. Firm size retains its larger-firm-positive orientation. Changes in attribution do not change the joint forecast.

Table IA.2: Actual stock portfolios with coefficient or stock-loading restrictions

Characteristic mandate	Exposure restriction	Loading-projection share (%)	Mean excess return (bp/month)	Return volatility (bp/month)	One-way turnover (% NAV/month)
Book-to-market	Coefficient orthogonality	46.10	24.49	181.03	13.05
Book-to-market	Stock-loading neutrality	0.00	12.93	83.09	18.40
Momentum	Coefficient orthogonality	53.51	42.99	251.31	23.81
Momentum	Stock-loading neutrality	0.00	21.91	93.30	35.71
Firm size	Coefficient orthogonality	65.13	9.23	205.45	9.87
Firm size	Stock-loading neutrality	0.00	-8.54	79.99	26.52

The table compares coefficient orthogonality with neutrality to estimated stock loadings at the same unit gross. It uses the unrestricted 153-characteristic source model with FFC6 and three latent factors. Both rules hold source stocks, impose zero net stock investment, and have unit gross holdings at formation. Coefficient orthogonality and current cash neutrality are imposed jointly. Loading-projection share is the fraction of squared stock-weight norm projected onto the joint estimated loading profiles, averaged over 225 formation months. A displayed 0.00 reflects the imposed stock-loading restriction and rounding, not zero future return risk. Mean and volatility use 216 common subsequent monthly excess returns, in basis points. One-way turnover is percent of NAV per month and uses return-adjusted prior holdings over 219 valid months. Firm size is larger-firm positive. Equal gross does not hold the desired characteristic exposure fixed.

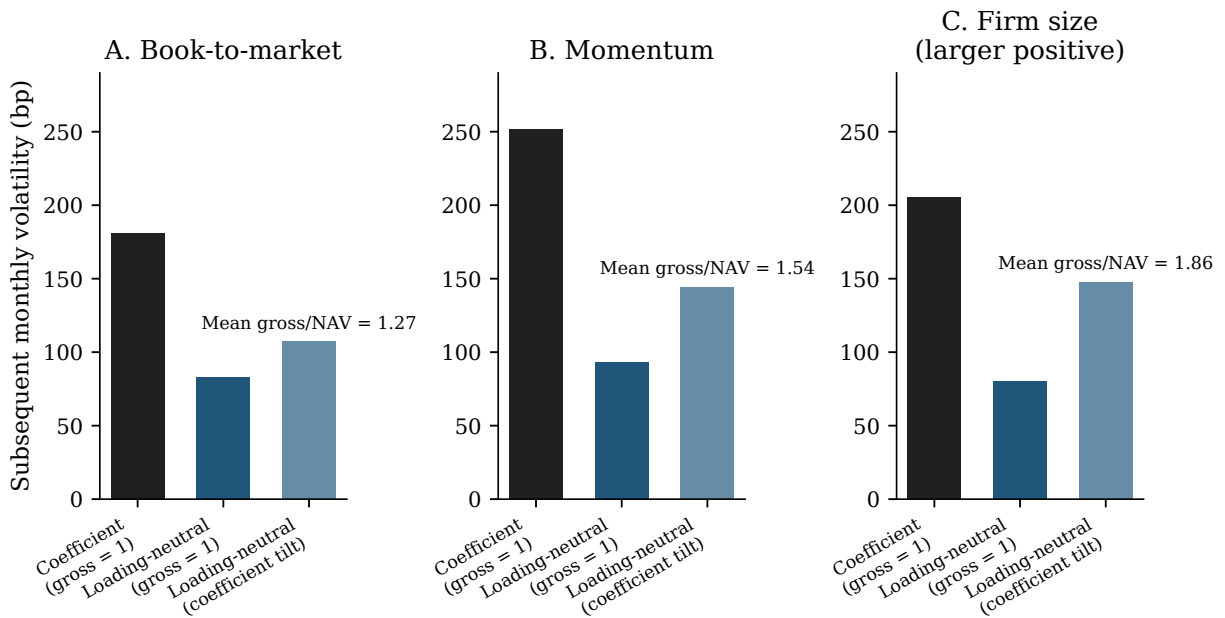


Figure IA.3: Stock-loading neutrality at unit gross and at the coefficient hedge's tilt

Lower bars indicate lower subsequent return volatility. The first two policies use unit gross; the third restores the coefficient hedge's signed characteristic exposure. The source model has 153 nonconstant characteristics, FFC6, three latent factors and a free characteristic intercept. The coefficient hedge and loading-neutral hedge use the same source stocks and annual loading estimates, impose zero net stock investment and are normalized to unit gross holdings. The third rule scales the loading-neutral holdings each month to match the coefficient hedge's signed characteristic tilt; gross holdings are then allowed to differ from one. The annotated gross positions are averages of the sum of absolute stock weights over 225 formation months. Bar heights are standard deviations of subsequent monthly excess returns over 216 common months, in basis points. The reference tilt is the coefficient hedge's, not the original unhedged portfolio's positive tilt used in the later covariance comparison. Native size is larger-firm positive.

Table IA.3: Training-average versus current characteristic exposures at unit gross

Source intercept / no. of characteristics	Characteristic mandate	Volatility (bp/month)		Training-rule loading share (%)	Current minus training	
		Training- average Q	Current Q		Exposure retention (pp)	One-way turnover (pp)
Unrestricted / 36	Book-to-market	97.47	77.25	4.70	-2.09	0.67
Unrestricted / 36	Momentum	120.00	89.68	6.58	-3.48	1.00
Unrestricted / 36	Firm size	133.59	88.24	9.52	10.68	0.56
Unrestricted / 153	Book-to-market	117.13	83.09	3.31	-0.24	0.44
Unrestricted / 153	Momentum	123.35	93.30	7.65	-4.60	0.81
Unrestricted / 153	Firm size	108.86	79.99	6.69	8.56	-0.31
Zero / 36	Book-to-market	96.07	76.86	4.47	-1.20	0.49
Zero / 36	Momentum	128.17	93.04	6.87	-2.98	1.30
Zero / 36	Firm size	139.00	87.98	10.27	10.44	0.35
Zero / 153	Book-to-market	115.46	81.07	3.46	0.01	0.22
Zero / 153	Momentum	120.78	94.80	7.30	-4.00	0.65
Zero / 153	Firm size	113.23	82.19	7.74	11.66	-0.52

The table compares historical and current exposure measurement while holding gross at one. Lower volatility means lower subsequent risk; the change columns show current minus training. All models use FFC6 plus three latent factors; model labels report the source characteristic-intercept restriction and library size. Both rules use the same annual loading matrix, current stocks, correction span and current zero-net-investment constraint, with unit gross holdings. Training-average Q is the mean of the preceding 240 formation matrices and remains fixed until the next annual fit; Current Q uses that month's stocks and characteristics. Volatility uses 216 common subsequent returns.

The training rule's loading-projection share is the fraction of squared stock-weight norm projected onto current stock-loading profiles; it is not a return-variance share. Exposure retention is measured relative to the original unhedged unit-gross portfolio. Both change columns subtract training from current; exposure uses 225 formation months and turnover 219 valid months. Firm size is larger-firm positive. These unit-gross entries differ from Panel A of Figure IA.4's matched signed-exposure comparison.

Table IA.4: Risk, gross holdings, and trading at the same positive characteristic exposure

Source intercept / no. of characteristics	Characteristic mandate	Historical minimum variance			Current- <i>Q</i> loading neutrality		
		Volatility (bp/month)	Gross holdings/NAV	One-way turnover (% NAV/month)	Volatility (bp/month)	Gross holdings/NAV	One-way turnover (% NAV/month)
Unrestricted / 36	Book-to-market	131.53	1.23	18.86	115.29	1.46	25.89
Unrestricted / 36	Momentum	240.14	1.42	45.31	204.26	2.34	81.40
Unrestricted / 36	Firm size	185.25	1.35	23.57	275.30	3.27	92.99
Unrestricted / 153	Book-to-market	141.18	1.21	20.03	112.47	1.32	24.56
Unrestricted / 153	Momentum	228.99	1.33	42.44	151.25	1.62	57.84
Unrestricted / 153	Firm size	171.25	1.28	24.86	152.12	1.92	51.29
Zero / 36	Book-to-market	133.94	1.24	19.14	115.09	1.47	27.62
Zero / 36	Momentum	252.19	1.42	44.15	213.32	2.42	83.17
Zero / 36	Firm size	179.75	1.34	22.57	273.32	3.27	91.69
Zero / 153	Book-to-market	142.76	1.21	19.73	109.24	1.32	24.26
Zero / 153	Momentum	228.61	1.33	42.97	158.28	1.70	59.38
Zero / 153	Firm size	172.05	1.27	24.55	152.74	1.89	49.30

The table compares historical variance minimization with current loading neutrality at the same economic target. Both rules hold the same positive signed characteristic exposure as the current unhedged unit-gross portfolio, impose zero net stock investment, and use the same span of that characteristic and the annual loading profiles within each model. Historical minimum variance uses complete characteristic-managed payoffs from the preceding 240 calendar months (209–233 observations), with covariance fixed between annual origins. Current-*Q* loading neutrality removes current estimated stock loadings and rescales to the common exposure. Neither rule is normalized afterward or constrained by gross or turnover caps. Volatility is in monthly basis points and uses 216 common subsequent months; gross is in multiples of NAV averaged over 225 formation months, and turnover is percent of NAV per month over 219 valid months. Firm size is larger-firm positive. Model labels denote source characteristic-intercept restriction and library size. Panel B of Figure IA.4 reports pointwise paired uncertainty.

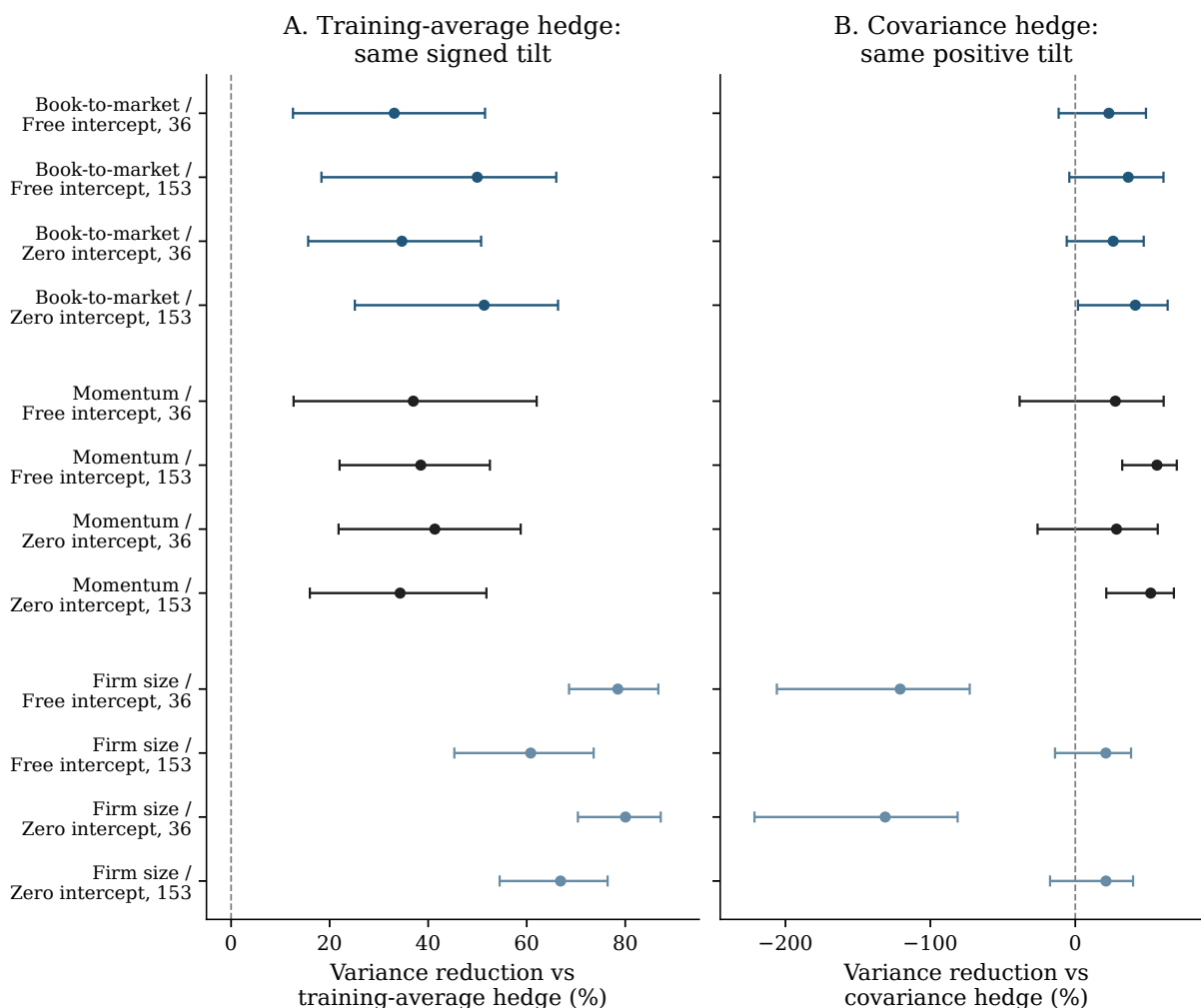


Figure IA.4: Current loading neutrality against two mandate-matched hedges
Each estimate is $100 \times [1 - \text{Var}(\text{current loading-neutral payoff}) / \text{Var}(\text{reference payoff})]$ over the same 216 subsequent-return months; positive values favor current loading neutrality. Panel A uses the frozen training-distribution hedge as the reference. Its characteristic second-moment matrix averages the preceding 240 formation months and is held until the next annual estimate. Current loading-neutral holdings are rescaled each month to the reference's signed characteristic tilt, including negative tilts. Panel B instead requires both current loading-neutral and historical covariance-minimizing holdings to retain the current unhedged unit-gross portfolio's positive characteristic tilt. Within each model they use the same current stocks, cash-neutral constraint and span of characteristic and annual loading profiles. Gross holdings and turnover are outcomes, not common constraints. All matching uses formation information over 225 months. Free and zero intercept identify the source characteristic-intercept restriction; 36 and 153 identify library size. Firm size is larger-firm positive, not SMB. Bars are pointwise 95% paired circular calendar-block intervals with 12-month blocks and 1,999 draws, conditional on the estimated models and evaluated paths. The two horizontal scales differ.

Table IA.5: Sources of changes in latent exposure coverage

Source intercept / no. of characteristics	Mean absolute coverage contribution (pp)		
	Loadings	Characteristic distribution	Latent-subspace distance (%)
Unrestricted / 36	1.92	0.27	6.08
Unrestricted / 153	1.08	0.24	8.88
Zero / 36	1.61	0.29	6.09
Zero / 153	1.39	0.24	13.89

The table separates changes in firms' characteristics from changes in estimated loadings as sources of changing exposure coverage. The first two columns are mean absolute contributions to changes in latent-space coverage of three fixed characteristics across 18 annual transitions. Symmetric averaging over the two orders of updating loadings and the characteristic distribution gives an exact decomposition before absolute values are taken. The displayed magnitudes are percentage-point movements, not shares of return variance or of a common signed total. Latent-subspace distance is the mean squared sine of principal angles under the previous reference Q , in percent. It describes the whole latent space and is invariant to relabeling individual factors. Model labels report the source characteristic-intercept restriction and library size.

IA.4 Complete Fixed-Positive Library Comparisons

The tables report all 24 policy levels and all twelve 153-minus-36 library comparisons. Each portfolio has zero net stock investment and matches the same original positive characteristic exposure. Free and zero refer to the source characteristic-intercept restriction, and firm size is larger-firm positive. Current- Q removes current stock-loading exposure; the covariance rule minimizes historical variance within the loading-plus-characteristic span. Returns and volatility are monthly basis points, gross is in NAV units, and one-way turnover is in percent. Gross holdings and turnover are time-series averages. The formation, return, and turnover counts are 225, 216, and 219 throughout.

Table IA.6 gives the simultaneous risk inference for the complete twelve-comparison library family. The volatility reduction is supported for loading-neutral size under both intercept restrictions and for loading-neutral momentum under the zero-intercept restriction. The other nine intervals include zero. The subsequent tables retain the original pointwise intervals for all reported metrics, allowing their magnitudes and uncertainty to be read without treating mean returns, gross, or turnover as members of the volatility family.

The constraints are checked at formation using achieved characteristic exposure and current stock-loading projection. These checks establish that the computed positions implement the specified rule. The economic comparisons concern the resulting return paths, gross holdings, and turnover.

Table IA.6: Expanding the library: all twelve simultaneous volatility comparisons

Source intercept	Characteristic	Rule	Volatility change (153 minus 36) (bp/month)	Familywise 95% CI (bp/month)	Adjusted p
Free	Book-to-market	Loading neutral	-2.82	[-27.67, 22.02]	0.99950
Free	Book-to-market	Covariance hedge	9.65	[-14.94, 34.23]	0.74990
Free	Momentum	Loading neutral	-53.00	[-107.19, 1.19]	0.05560
Free	Momentum	Covariance hedge	-11.14	[-47.22, 24.93]	0.88330
Free	Larger-firm size	Loading neutral	-123.18	[-166.54, -79.81]	0.00005
Free	Larger-firm size	Covariance hedge	-13.99	[-61.68, 33.69]	0.90475
Zero	Book-to-market	Loading neutral	-5.85	[-27.37, 15.66]	0.92915
Zero	Book-to-market	Covariance hedge	8.82	[-17.52, 35.16]	0.84615
Zero	Momentum	Loading neutral	-55.04	[-99.43, -10.65]	0.01475
Zero	Momentum	Covariance hedge	-23.58	[-57.58, 10.43]	0.24070
Zero	Larger-firm size	Loading neutral	-120.58	[-172.92, -68.23]	0.00015
Zero	Larger-firm size	Covariance hedge	-7.71	[-55.56, 40.15]	0.99460

Each row compares policies maintaining the same original positive characteristic target on the same formation stocks.

Entries are monthly volatility under 153 characteristics minus monthly volatility under 36 characteristics, in basis points; negative values favor the larger library. Three intervals exclude zero: loading-neutral size under both intercept restrictions and loading-neutral momentum under the zero-intercept restriction. The family comprises three positive characteristic mandates, two source-intercept restrictions, and two portfolio rules. Free and Zero refer to the source characteristic intercept; size means larger firms. Simultaneous 95% intervals and adjusted p -values use 19,999 shared twelve-month circular calendar-block draws and the studentized maximum absolute t statistic over these twelve comparisons. Inference is conditional on the fitted annual models and uses the original 216 common return months. It applies to this volatility family separately from the two current-versus-origin variance families; mean returns, gross holdings, and turnover are not covered by these intervals.

Table IA.7: Original positive target: unrestricted-intercept portfolio outcomes

Characteristic	No. of char. / rule	Mean excess return (bp/month)	Volatility (bp/month)	Mean gross / NAV	One-way turnover (% NAV/month)	Target exposure
Book-to-market	36 / Q	18.32	115.29	1.458	25.89	0.334
Book-to-market	36 / Cov	24.70	131.53	1.235	18.86	0.334
Book-to-market	153 / Q	17.03	112.47	1.323	24.56	0.334
Book-to-market	153 / Cov	25.52	141.18	1.214	20.03	0.334
Momentum	36 / Q	18.30	204.26	2.338	81.40	0.333
Momentum	36 / Cov	15.63	240.14	1.424	45.31	0.333
Momentum	153 / Q	34.42	151.25	1.620	57.84	0.333
Momentum	153 / Cov	34.47	228.99	1.326	42.44	0.333
Larger-firm size	36 / Q	12.85	275.30	3.272	92.99	0.334
Larger-firm size	36 / Cov	-7.65	185.25	1.346	23.57	0.334
Larger-firm size	153 / Q	-15.66	152.12	1.923	51.29	0.334
Larger-firm size	153 / Cov	-10.94	171.25	1.283	24.86	0.334

Each policy preserves the same original positive characteristic target and cash neutrality on the formation stocks. Q denotes Current loading neutrality; Cov denotes the historical covariance hedge. Current removes current stock-loading exposure; Covariance minimizes historical variance within the loading-plus-characteristic span. The two libraries use separately estimated loading spaces. Returns and volatility are monthly basis points over 216 common return months; gross is mean absolute holdings divided by NAV over 225 formation months; one-way turnover is percent of NAV per month over 219 eligible months. Size is larger-firm positive. These are descriptive levels; the following tables provide the separate uncertainty comparisons.

Table IA.8: Original positive target: zero-intercept portfolio outcomes

Characteristic	No. of char. / rule	Mean excess return (bp/month)	Volatility (bp/month)	Mean gross / NAV	One-way turnover (% NAV/month)	Target exposure
Book-to-market	36 / Q	6.23	115.09	1.468	27.62	0.334
Book-to-market	36 / Cov	25.35	133.94	1.239	19.14	0.334
Book-to-market	153 / Q	4.79	109.24	1.324	24.26	0.334
Book-to-market	153 / Cov	25.48	142.76	1.211	19.73	0.334
Momentum	36 / Q	-24.74	213.32	2.416	83.17	0.333
Momentum	36 / Cov	3.94	252.19	1.422	44.15	0.333
Momentum	153 / Q	-1.60	158.28	1.702	59.38	0.333
Momentum	153 / Cov	22.25	228.61	1.329	42.97	0.333
Larger-firm size	36 / Q	31.60	273.32	3.265	91.69	0.334
Larger-firm size	36 / Cov	-11.92	179.75	1.343	22.57	0.334
Larger-firm size	153 / Q	-11.00	152.74	1.893	49.30	0.334
Larger-firm size	153 / Cov	-11.01	172.05	1.267	24.55	0.334

Each policy preserves the same original positive characteristic target and cash neutrality on the formation stocks. Q denotes Current loading neutrality; Cov denotes the historical covariance hedge. Current removes current stock-loading exposure; Covariance minimizes historical variance within the loading-plus-characteristic span. The two libraries use separately estimated loading spaces. Returns and volatility are monthly basis points over 216 common return months; gross is mean absolute holdings divided by NAV over 225 formation months; one-way turnover is percent of NAV per month over 219 eligible months. Size is larger-firm positive. These are descriptive levels; the following tables provide the separate uncertainty comparisons.

Table IA.9: Expanding the library under Current loading neutrality: original pointwise comparisons

	Mean excess return change (bp/month)	Volatility change (bp/month)	Mean gross change (NAV units)	One-way turnover change (pp/month)
<i>Free source characteristic intercept</i>				
Book-to-market	-1.29 [-10.56,9.46]	-2.82 [-14.95,9.90]	-0.135 [-0.151,-0.119]	-1.32 [-2.01,-0.69]
Momentum	16.12 [-7.18,36.28]	-53.00 [-76.69,-23.71]	-0.718 [-0.764,-0.676]	-23.57 [-25.71,-21.57]
Larger-firm size	-28.52 [-62.40,7.36]	-123.18 [-143.76,-99.57]	-1.349 [-1.479,-1.223]	-41.70 [-46.02,-37.23]
<i>Zero source characteristic intercept</i>				
Book-to-market	-1.44 [-11.21,10.26]	-5.85 [-16.91,5.42]	-0.144 [-0.167,-0.124]	-3.36 [-4.30,-2.51]
Momentum	23.13 [1.57,45.36]	-55.04 [-75.58,-31.41]	-0.714 [-0.776,-0.656]	-23.79 [-25.24,-22.23]
Larger-firm size	-42.60 [-77.92,-6.49]	-120.58 [-146.43,-92.98]	-1.372 [-1.493,-1.248]	-42.39 [-47.76,-36.94]

Entries are the 153-characteristic outcome minus the 36-characteristic outcome at the same original positive target. Negative volatility changes mean less subsequent risk; negative gross and turnover changes mean smaller positions and less trading. Mean excess return and volatility are monthly basis points, gross is in NAV multiples, and turnover is in percentage points of NAV per month. Returns use 216 common months, gross 225 formation months, and turnover 219 eligible months. Brackets retain the original pointwise 95% paired conditional calendar-block intervals, using the same 1,999 twelve-month circular draws. Free and Zero identify source intercepts; size is larger-firm positive. For the separate simultaneous volatility inference, see Table IA.6.

Table IA.10: Expanding the library under historical covariance hedging: original pointwise comparisons

	Mean excess return change (bp/month)	Volatility change (bp/month)	Mean gross change (NAV units)	One-way turnover change (pp/month)
<i>Free source characteristic intercept</i>				
	0.82	9.65	-0.020	1.17
Book-to-market	[-10.40,11.81]	[-2.30,21.87]	[-0.028,-0.012]	[0.50,1.77]
	18.84	-11.14	-0.098	-2.86
Momentum	[2.72,32.85]	[-28.13,6.99]	[-0.115,-0.080]	[-4.53,-1.15]
	-3.29	-13.99	-0.062	1.29
Larger-firm size	[-24.38,15.78]	[-36.28,9.25]	[-0.086,-0.040]	[-0.08,2.49]
<i>Zero source characteristic intercept</i>				
	0.14	8.82	-0.027	0.59
Book-to-market	[-11.16,10.56]	[-4.37,21.96]	[-0.039,-0.017]	[-0.39,1.33]
	18.31	-23.58	-0.092	-1.18
Momentum	[6.07,31.88]	[-39.58,-5.09]	[-0.124,-0.056]	[-3.05,0.68]
	0.91	-7.71	-0.076	1.98
Larger-firm size	[-15.47,18.47]	[-28.51,17.89]	[-0.101,-0.051]	[0.82,3.02]

Entries are the 153-characteristic outcome minus the 36-characteristic outcome at the same original positive target. Negative volatility changes mean less subsequent risk; negative gross and turnover changes mean smaller positions and less trading. Mean excess return and volatility are monthly basis points, gross is in NAV multiples, and turnover is in percentage points of NAV per month. Returns use 216 common months, gross 225 formation months, and turnover 219 eligible months. Brackets retain the original pointwise 95% paired conditional calendar-block intervals, using the same 1,999 twelve-month circular draws. Free and Zero identify source intercepts; size is larger-firm positive. For the separate simultaneous volatility inference, see Table IA.6.

Table IA.11: Expanding the library: variance reduction relative to the smaller model

Source intercept	Characteristic	Rule	Variance reduction 153 vs. 36 (%)	Lower 95% endpoint	Upper 95% endpoint
Free	Book-to-market	Q	4.84	-19.65	23.43
Free	Book-to-market	Cov	-15.21	-36.60	3.80
Free	Momentum	Q	45.16	25.78	56.47
Free	Momentum	Cov	9.07	-8.28	20.33
Free	Larger-firm size	Q	69.47	63.30	74.04
Free	Larger-firm size	Cov	14.53	-11.26	34.30
Zero	Book-to-market	Q	9.91	-10.28	26.64
Zero	Book-to-market	Cov	-13.60	-34.02	7.14
Zero	Momentum	Q	44.94	31.82	54.10
Zero	Momentum	Cov	17.82	5.00	26.19
Zero	Larger-firm size	Q	68.77	60.68	74.51
Zero	Larger-firm size	Cov	8.39	-21.96	28.84

Entries are percentage reductions in subsequent variance under 153 characteristics relative to the corresponding 36-characteristic policy, at the same original positive target. Positive values favor the larger library. Q denotes Current loading neutrality; Cov denotes the historical covariance hedge. Each denominator is the variance of that row's 36-characteristic policy, so reductions across rules have different denominators. Lower and upper columns give original pointwise 95% conditional paired intervals over the common return sample; they are not the simultaneous volatility intervals. Free and Zero identify source characteristic intercepts, and size is larger-firm positive.

Table IA.12: Loading overlap, residual stock positions, and required gross holdings

Source intercept	Characteristic	Cash-adjusted coverage: 36 (%)	Cash-adjusted coverage: 153 (%)	Loading overlap contribution	Residual stock shape contribution	Log gross 153 minus 36 change
Free	Book-to-market	58.71	47.38	-0.2431	0.1464	-0.0967
Free	Momentum	83.48	65.14	-0.7455	0.3791	-0.3665
Free	Larger-firm size	91.84	76.27	-1.0926	0.5674	-0.5252
Zero	Book-to-market	59.08	47.43	-0.2531	0.1502	-0.1028
Zero	Momentum	84.60	68.47	-0.7206	0.3717	-0.3490
Zero	Larger-firm size	91.72	75.22	-1.1189	0.5797	-0.5392

The table connects the desired characteristic profile to the scale of the selected Current projection portfolio. Coverage uses the current stock distribution and the loading space including cash. Gross is in multiples of NAV; lower gross means smaller absolute stock positions for the same original positive target. Entries average all 225 monthly values within each policy. The three log columns are changes from 36 to 153 characteristics: the overlap contribution and residual-position-shape contribution sum to the change in log gross before rounding. The three log columns average monthly paired terms before aggregation, not logarithms of sample-average gross. They are descriptive accounting quantities for the selected direction, not a minimum-gross solution or a decomposition of subsequent return variance. Free and Zero denote source characteristic intercepts; size is larger-firm positive. Figure 5 displays the same terms.

IA.5 Complete Three-Reference Comparisons

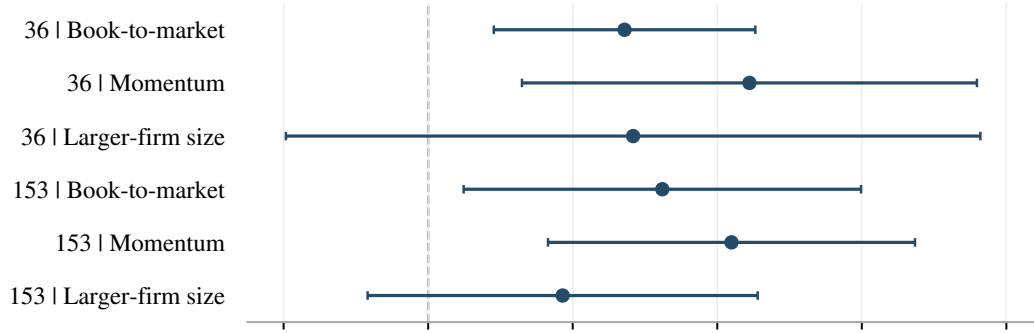
Two normalizations answer different investment questions. Unit gross devotes the same absolute stock position to each rule, allowing achieved characteristic exposures to differ. The signed training reference instead matches the training-reference unit-gross portfolio's actual monthly exposure, including negative months. This target is common across references within each model, but can differ across fitted libraries. Every rule uses current stocks, the same annual loading coefficients, and current cash neutrality. Current measurement also enforces current loading neutrality; historical references generally leave some current loading exposure.

Figure IA.5 and Table IA.13 report the twelve current–origin variance comparisons with familywise intervals. Eight intervals exclude zero, covering book-to-market and momentum in both libraries and intercept specifications; the four size intervals include zero. These intervals use 19,999 shared calendar-block draws and apply to this signed-target family alone. The following complete tables retain the original 1,999-draw pointwise intervals for all three reference contrasts and both normalizations. Those pointwise intervals answer individual-comparison questions and do not carry the simultaneous coverage of Table IA.13.

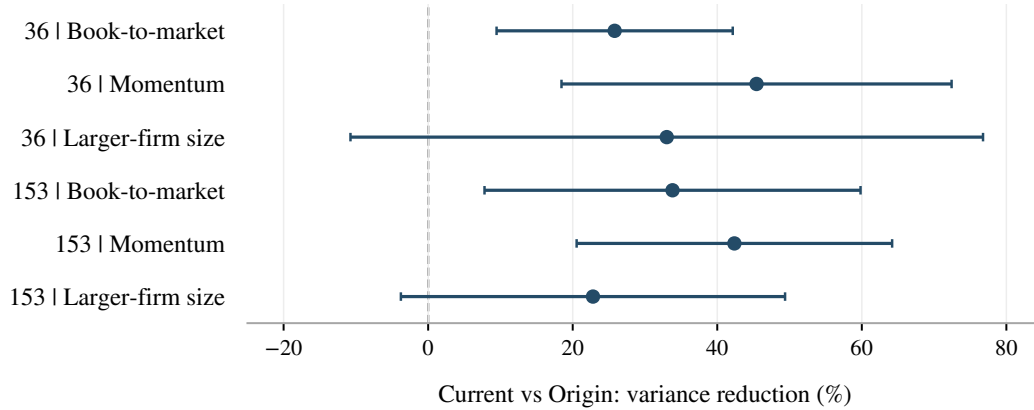
The complete tables preserve all twelve source-model–characteristic combinations for each normalization and reference comparison. The accompanying results files retain all 576 original pointwise interval rows, including achieved-exposure and loading-projection diagnostics, together with the separate familywise results. Different reference variances prevent adding the two sequential percentage reductions. These comparisons therefore distinguish the information update from the normalization used to translate it into stock positions.

Signed characteristic target inherited from Training

A. Free source characteristic intercept



B. Zero source characteristic intercept



Positive values: lower subsequent variance under Current.

Points: estimates. Bars: simultaneous 95% intervals for this 12-comparison family.

Figure IA.5: Current versus annual-origin variance at the training rule's signed target

Both rules match the signed exposure of the training-reference unit-gross portfolio in each month, including negative targets. This differs from the original positive mandate in Figure 6. Positive values indicate lower subsequent return variance under current measurement. Points are $100[1 - \text{Var}(r^{\text{current}}) / \text{Var}(r^{\text{origin}})]$ over 216 common return months. Lines are familywise 95% intervals for these twelve signed-target comparisons, using 19,999 common twelve-month circular calendar-block draws and studentized maximum absolute statistics. All eight book-to-market and momentum intervals exclude zero; the four size intervals include zero. Inference conditions on the fitted models and is separate from the positive-target and library-change families. The rules use the same current stocks, annual loadings, and cash constraint; their contemporaneous loading restrictions differ. Free and Zero denote source-intercept restrictions; 36 and 153 count characteristics, and Size denotes the larger-firm characteristic. Every rule forms holdings monthly.

Table IA.13: Current versus Origin: variance at the Training rule's signed target

Source intercept / no. of characteristics	Characteristic	Variance reduction vs. Origin (%)	Familywise 95% CI (%)	Adjusted p
Free / 36	Book-to-market	27.18	[9.08, 45.27]	0.00445
Free / 36	Momentum	44.45	[12.98, 75.93]	0.00650
Free / 36	Larger-firm size	28.36	[-19.68, 76.40]	0.33500
Free / 153	Book-to-market	32.40	[4.91, 59.89]	0.02105
Free / 153	Momentum	41.97	[16.58, 67.35]	0.00225
Free / 153	Larger-firm size	18.61	[-8.38, 45.59]	0.21555
Zero / 36	Book-to-market	25.80	[9.47, 42.14]	0.00310
Zero / 36	Momentum	45.44	[18.45, 72.42]	0.00200
Zero / 36	Larger-firm size	33.01	[-10.75, 76.77]	0.16060
Zero / 153	Book-to-market	33.81	[7.80, 59.82]	0.01150
Zero / 153	Momentum	42.37	[20.54, 64.20]	0.00075
Zero / 153	Larger-firm size	22.80	[-3.77, 49.38]	0.09945

Each pair matches the Training unit-gross portfolio's signed characteristic exposure within the month; the target can be negative and differs from the original positive mandate. Entries measure the percentage reduction in Current return variance relative to Origin, so positive values favor Current. Eight of twelve simultaneous intervals exclude zero: all book-to-market and momentum comparisons; all four size intervals include zero. The estimate is $100[1 - \text{Var}(r^{\text{current}}) / \text{Var}(r^{\text{origin}})]$. Brackets and adjusted p -values use this separate twelve-member family, 19,999 common calendar-block draws, and the studentized maximum absolute statistic, conditional on frozen fitted models. All comparisons use 216 common return months, the same annual loading coefficients, formation stocks and current cash constraint. Their current loading restrictions differ. Free and Zero identify source characteristic intercepts; size means a positive larger-firm profile, even when the inherited signed target is negative.

Table IA.14: Signed training-reference exposure: complete unrestricted-source-intercept levels

No. of char.	Characteristic	Reference	Mean excess return (bp/month)	Volatility (bp/month)	Mean gross / NAV	One-way turnover (% NAV/month)
36	Book-to-market	Train	11.13	97.47	1.000	16.92
36	Book-to-market	Origin	13.34	93.41	1.046	18.61
36	Book-to-market	Current	12.87	79.71	1.031	18.16
36	Momentum	Train	9.32	120.00	1.000	33.82
36	Momentum	Origin	4.57	127.82	1.066	37.39
36	Momentum	Current	5.79	95.26	1.074	38.26
36	Larger-firm size	Train	-7.73	133.59	1.000	27.72
36	Larger-firm size	Origin	7.10	73.20	0.737	22.04
36	Larger-firm size	Current	6.23	61.96	0.679	20.79
153	Book-to-market	Train	13.33	117.13	1.000	17.96
153	Book-to-market	Origin	11.09	100.78	1.019	18.87
153	Book-to-market	Current	13.46	82.86	1.003	18.47
153	Momentum	Train	21.40	123.35	1.000	34.90
153	Momentum	Origin	22.61	126.99	1.066	37.70
153	Momentum	Current	23.42	96.74	1.075	38.31
153	Larger-firm size	Train	-17.73	108.86	1.000	26.83
153	Larger-firm size	Origin	-8.86	75.54	0.841	22.50
153	Larger-firm size	Current	-7.68	68.15	0.831	22.25

All references match the Training unit-gross portfolio's signed characteristic exposure in each month, including negative months. Each rule uses the same current stocks, annual loading coefficients and cash constraint; Current is current-loading neutral, while historical references generally retain current loading exposure. Mean excess return and volatility are monthly basis points over 216 common months. Gross is mean absolute stock holdings divided by NAV over 225 formation months; turnover is one-way percent of NAV per month over 219 eligible months. Size is larger-firm positive. These are descriptive policy levels.

Table IA.15: Signed training-reference exposure: complete zero-source-intercept levels

No. of char.	Characteristic	Reference	Mean excess return (bp/month)	Volatility (bp/month)	Mean gross / NAV	One-way turnover (% NAV/month)
36	Book-to-market	Train	2.96	96.07	1.000	18.17
36	Book-to-market	Origin	5.63	90.15	1.035	19.59
36	Book-to-market	Current	4.21	77.65	1.018	18.99
36	Momentum	Train	-15.10	128.17	1.000	33.17
36	Momentum	Origin	-12.89	132.88	1.064	36.96
36	Momentum	Current	-11.37	98.15	1.067	37.51
36	Larger-firm size	Train	-0.37	139.00	1.000	27.45
36	Larger-firm size	Origin	7.86	75.86	0.746	22.09
36	Larger-firm size	Current	7.48	62.09	0.699	20.92
153	Book-to-market	Train	7.01	115.46	1.000	17.97
153	Book-to-market	Origin	2.52	98.97	1.013	18.54
153	Book-to-market	Current	4.05	80.52	1.000	18.22
153	Momentum	Train	-1.54	120.78	1.000	34.28
153	Momentum	Origin	-2.28	128.95	1.075	36.98
153	Momentum	Current	0.01	97.89	1.070	37.24
153	Larger-firm size	Train	-19.99	113.23	1.000	26.39
153	Larger-firm size	Origin	-8.20	74.18	0.793	20.85
153	Larger-firm size	Current	-5.44	65.18	0.778	20.48

All references match the Training unit-gross portfolio's signed characteristic exposure in each month, including negative months. Each rule uses the same current stocks, annual loading coefficients and cash constraint; Current is current-loading neutral, while historical references generally retain current loading exposure. Mean excess return and volatility are monthly basis points over 216 common months. Gross is mean absolute stock holdings divided by NAV over 225 formation months; turnover is one-way percent of NAV per month over 219 eligible months. Size is larger-firm positive. These are descriptive policy levels.

Table IA.16: Unit gross: complete unrestricted-source-intercept levels

No. of char.	Characteristic	Reference	Mean excess return (bp/month)	Volatility (bp/month)	Mean gross / NAV	One-way turnover (% NAV/month)
36	Book-to-market	Train	11.13	97.47	1.000	16.92
36	Book-to-market	Origin	12.42	88.89	1.000	17.78
36	Book-to-market	Current	12.48	77.25	1.000	17.59
36	Momentum	Train	9.32	120.00	1.000	33.82
36	Momentum	Origin	7.01	127.54	1.000	34.65
36	Momentum	Current	6.58	89.68	1.000	34.82
36	Larger-firm size	Train	-7.73	133.59	1.000	27.72
36	Larger-firm size	Origin	2.65	105.20	1.000	28.46
36	Larger-firm size	Current	5.76	88.24	1.000	28.28
153	Book-to-market	Train	13.33	117.13	1.000	17.96
153	Book-to-market	Origin	10.20	98.87	1.000	18.49
153	Book-to-market	Current	12.93	83.09	1.000	18.40
153	Momentum	Train	21.40	123.35	1.000	34.90
153	Momentum	Origin	21.31	124.55	1.000	35.43
153	Momentum	Current	21.91	93.30	1.000	35.71
153	Larger-firm size	Train	-17.73	108.86	1.000	26.83
153	Larger-firm size	Origin	-10.11	91.47	1.000	26.74
153	Larger-firm size	Current	-8.54	79.99	1.000	26.52

All references allocate one unit of gross to stocks, allowing achieved characteristic exposures to differ. Each rule uses the same current stocks, annual loading coefficients and cash constraint; Current is current-loading neutral, while historical references generally retain current loading exposure. Mean excess return and volatility are monthly basis points over 216 common months. Gross is mean absolute stock holdings divided by NAV over 225 formation months; turnover is one-way percent of NAV per month over 219 eligible months. Size is larger-firm positive. These are descriptive policy levels.

Table IA.17: Unit gross: complete zero-source-intercept levels

No. of char.	Characteristic	Reference	Mean excess return (bp/month)	Volatility (bp/month)	Mean gross / NAV	One-way turnover (% NAV/month)
36	Book-to-market	Train	2.96	96.07	1.000	18.17
36	Book-to-market	Origin	5.34	87.37	1.000	18.96
36	Book-to-market	Current	4.29	76.86	1.000	18.66
36	Momentum	Train	-15.10	128.17	1.000	33.17
36	Momentum	Origin	-10.87	137.57	1.000	34.37
36	Momentum	Current	-11.26	93.04	1.000	34.47
36	Larger-firm size	Train	-0.37	139.00	1.000	27.45
36	Larger-firm size	Origin	6.05	106.18	1.000	27.75
36	Larger-firm size	Current	11.16	87.98	1.000	27.79
153	Book-to-market	Train	7.01	115.46	1.000	17.97
153	Book-to-market	Origin	2.29	97.71	1.000	18.28
153	Book-to-market	Current	3.70	81.07	1.000	18.19
153	Momentum	Train	-1.54	120.78	1.000	34.28
153	Momentum	Origin	-1.88	126.29	1.000	34.53
153	Momentum	Current	0.56	94.80	1.000	34.93
153	Larger-firm size	Train	-19.99	113.23	1.000	26.39
153	Larger-firm size	Origin	-9.64	95.35	1.000	26.15
153	Larger-firm size	Current	-6.62	82.19	1.000	25.87

All references allocate one unit of gross to stocks, allowing achieved characteristic exposures to differ. Each rule uses the same current stocks, annual loading coefficients and cash constraint; Current is current-loading neutral, while historical references generally retain current loading exposure. Mean excess return and volatility are monthly basis points over 216 common months. Gross is mean absolute stock holdings divided by NAV over 225 formation months; turnover is one-way percent of NAV per month over 219 eligible months. Size is larger-firm positive. These are descriptive policy levels.

Table IA.18: Signed training-reference exposure: origin vs training (risk and mean return)

Source intercept / no. of characteristics	Characteristic	Variance reduction (%)	Mean excess return change (bp/month)
Free / 36	Book-to-market	8.16 [-18.90,32.34]	2.21 [-8.06,11.80]
Free / 36	Momentum	-13.45 [-54.28,22.17]	-4.75 [-14.98,5.86]
Free / 36	Larger-firm size	69.98 [57.27,78.20]	14.84 [-3.99,36.14]
Free / 153	Book-to-market	25.97 [-1.93,42.34]	-2.25 [-16.88,9.16]
Free / 153	Momentum	-5.98 [-51.07,27.81]	1.22 [-10.02,12.44]
Free / 153	Larger-firm size	51.85 [37.13,63.26]	8.87 [1.65,15.64]
Zero / 36	Book-to-market	11.94 [-14.03,34.56]	2.67 [-6.94,12.51]
Zero / 36	Momentum	-7.48 [-39.95,18.50]	2.21 [-8.67,12.71]
Zero / 36	Larger-firm size	70.22 [57.31,78.24]	8.23 [-11.07,28.48]
Zero / 153	Book-to-market	26.52 [3.14,41.25]	-4.49 [-18.00,6.26]
Zero / 153	Momentum	-13.99 [-55.37,19.73]	-0.74 [-12.73,11.06]
Zero / 153	Larger-firm size	57.08 [46.29,65.27]	11.80 [1.97,20.41]

The policies preserve the Training rule's signed characteristic exposure, including negative targets. Variance reduction is relative to Training: positive values mean lower variance under Origin. Mean excess return and volatility differences are Origin minus Training, in monthly basis points. All rules use the same formation stocks, annual loading coefficients and current cash constraint, but their current loading restrictions differ. Brackets contain pointwise 95% paired twelve-month circular calendar-block intervals (1,999 draws), conditional on frozen models and evaluated paths. Return quantities use 216 common months; gross uses 225 formation months and one-way turnover 219 eligible months. Free and Zero identify source characteristic intercepts; size is larger-firm positive.

Table IA.19: Signed training-reference exposure: origin vs training (implementation)

Source intercept / no. of characteristics	Characteristic	Mean gross change (NAV units)	One-way turnover change (pp/month)
Free / 36	Book-to-market	0.046 [0.025,0.065]	1.69 [0.95,2.39]
Free / 36	Momentum	0.066 [-0.030,0.147]	3.57 [0.49,6.24]
Free / 36	Larger-firm size	-0.263 [-0.387,-0.149]	-5.67 [-8.58,-2.75]
Free / 153	Book-to-market	0.019 [0.003,0.035]	0.91 [0.35,1.47]
Free / 153	Momentum	0.066 [0.035,0.099]	2.80 [1.15,4.39]
Free / 153	Larger-firm size	-0.159 [-0.207,-0.112]	-4.33 [-5.98,-2.74]
Zero / 36	Book-to-market	0.035 [0.014,0.054]	1.42 [0.67,2.13]
Zero / 36	Momentum	0.064 [-0.020,0.145]	3.79 [1.33,6.19]
Zero / 36	Larger-firm size	-0.254 [-0.372,-0.135]	-5.35 [-8.36,-2.24]
Zero / 153	Book-to-market	0.013 [-0.003,0.030]	0.56 [-0.03,1.20]
Zero / 153	Momentum	0.075 [0.032,0.121]	2.70 [1.03,4.48]
Zero / 153	Larger-firm size	-0.207 [-0.257,-0.158]	-5.54 [-7.23,-3.99]

The policies preserve the Training rule's signed characteristic exposure, including negative targets. Entries are Origin minus Training. Negative gross or turnover differences mean smaller positions or less trading under Origin; they do not indicate a return gain. Gross changes are NAV multiples and one-way turnover changes are percentage points of NAV per month. All rules use the same formation stocks, annual loading coefficients and current cash constraint, but their current loading restrictions differ. Brackets contain pointwise 95% paired twelve-month circular calendar-block intervals (1,999 draws), conditional on frozen models and evaluated paths. Return quantities use 216 common months; gross uses 225 formation months and one-way turnover 219 eligible months. Free and Zero identify source characteristic intercepts; size is larger-firm positive.

Table IA.20: Signed training-reference exposure: current vs origin (risk and mean return)

Source intercept / no. of characteristics	Characteristic	Variance reduction (%)	Mean excess return change (bp/month)
Free / 36	Book-to-market	27.18 [17.57,35.09]	-0.47 [-6.33,5.83]
Free / 36	Momentum	44.45 [28.47,60.37]	1.22 [-6.77,9.33]
Free / 36	Larger-firm size	28.36 [2.52,48.93]	-0.88 [-8.38,5.51]
Free / 153	Book-to-market	32.40 [17.00,43.46]	2.37 [-6.05,11.52]
Free / 153	Momentum	41.97 [27.75,52.31]	0.80 [-6.74,8.27]
Free / 153	Larger-firm size	18.61 [5.06,31.58]	1.18 [-2.65,5.06]
Zero / 36	Book-to-market	25.80 [17.98,33.03]	-1.42 [-7.32,4.55]
Zero / 36	Momentum	45.44 [28.23,57.88]	1.52 [-8.61,11.46]
Zero / 36	Larger-firm size	33.01 [9.57,53.24]	-0.38 [-8.61,6.00]
Zero / 153	Book-to-market	33.81 [20.05,44.82]	1.52 [-6.43,10.26]
Zero / 153	Momentum	42.37 [31.35,53.97]	2.29 [-7.11,11.20]
Zero / 153	Larger-firm size	22.80 [9.40,35.79]	2.76 [-1.36,7.20]

The policies preserve the Training rule's signed characteristic exposure, including negative targets. Variance reduction is relative to Origin: positive values mean lower variance under Current. Mean excess return and volatility differences are Current minus Origin, in monthly basis points. All rules use the same formation stocks, annual loading coefficients and current cash constraint, but their current loading restrictions differ. Brackets contain pointwise 95% paired twelve-month circular calendar-block intervals (1,999 draws), conditional on frozen models and evaluated paths. Return quantities use 216 common months; gross uses 225 formation months and one-way turnover 219 eligible months. Free and Zero identify source characteristic intercepts; size is larger-firm positive.

Table IA.21: Signed training-reference exposure: current vs origin (implementation)

Source intercept / no. of characteristics	Characteristic	Mean gross change (NAV units)	One-way turnover change (pp/month)
Free / 36	Book-to-market	-0.015 [-0.031,0.000]	-0.45 [-0.91,-0.00]
Free / 36	Momentum	0.008 [-0.066,0.080]	0.87 [-1.67,3.56]
Free / 36	Larger-firm size	-0.058 [-0.106,-0.009]	-1.25 [-2.87,0.27]
Free / 153	Book-to-market	-0.016 [-0.030,-0.004]	-0.39 [-0.77,-0.05]
Free / 153	Momentum	0.009 [-0.014,0.035]	0.61 [-0.74,2.06]
Free / 153	Larger-firm size	-0.010 [-0.038,0.017]	-0.25 [-1.26,0.69]
Zero / 36	Book-to-market	-0.017 [-0.030,-0.004]	-0.60 [-1.04,-0.13]
Zero / 36	Momentum	0.002 [-0.084,0.085]	0.56 [-2.57,3.55]
Zero / 36	Larger-firm size	-0.048 [-0.093,-0.002]	-1.17 [-3.06,0.49]
Zero / 153	Book-to-market	-0.013 [-0.025,-0.003]	-0.32 [-0.66,-0.02]
Zero / 153	Momentum	-0.005 [-0.039,0.030]	0.27 [-1.19,1.83]
Zero / 153	Larger-firm size	-0.015 [-0.040,0.008]	-0.37 [-1.22,0.41]

The policies preserve the Training rule's signed characteristic exposure, including negative targets. Entries are Current minus Origin. Negative gross or turnover differences mean smaller positions or less trading under Current; they do not indicate a return gain. Gross changes are NAV multiples and one-way turnover changes are percentage points of NAV per month. All rules use the same formation stocks, annual loading coefficients and current cash constraint, but their current loading restrictions differ. Brackets contain pointwise 95% paired twelve-month circular calendar-block intervals (1,999 draws), conditional on frozen models and evaluated paths. Return quantities use 216 common months; gross uses 225 formation months and one-way turnover 219 eligible months. Free and Zero identify source characteristic intercepts; size is larger-firm positive.

Table IA.22: Signed training-reference exposure: current vs training (risk and mean return)

Source intercept / no. of characteristics	Characteristic	Variance reduction (%)	Mean excess return change (bp/month)
Free / 36	Book-to-market	33.12 [12.56,51.54]	1.74 [-8.48,13.95]
Free / 36	Momentum	36.98 [12.68,62.03]	-3.52 [-11.80,4.69]
Free / 36	Larger-firm size	78.49 [68.59,86.71]	13.96 [-2.65,31.34]
Free / 153	Book-to-market	49.95 [18.33,66.00]	0.12 [-11.85,13.09]
Free / 153	Momentum	38.50 [22.02,52.53]	2.02 [-9.57,14.31]
Free / 153	Larger-firm size	60.81 [45.31,73.59]	10.04 [2.48,17.22]
Zero / 36	Book-to-market	34.66 [15.64,50.75]	1.25 [-8.47,12.81]
Zero / 36	Momentum	41.35 [21.81,58.78]	3.73 [-4.56,12.66]
Zero / 36	Larger-firm size	80.05 [70.39,87.20]	7.85 [-10.28,25.10]
Zero / 153	Book-to-market	51.37 [25.12,66.36]	-2.97 [-13.77,8.94]
Zero / 153	Momentum	34.31 [15.98,51.82]	1.55 [-8.04,11.56]
Zero / 153	Larger-firm size	66.87 [54.51,76.42]	14.55 [4.40,23.75]

The policies preserve the Training rule's signed characteristic exposure, including negative targets. Variance reduction is relative to Training: positive values mean lower variance under Current. Mean excess return and volatility differences are Current minus Training, in monthly basis points. All rules use the same formation stocks, annual loading coefficients and current cash constraint, but their current loading restrictions differ. Brackets contain pointwise 95% paired twelve-month circular calendar-block intervals (1,999 draws), conditional on frozen models and evaluated paths. Return quantities use 216 common months; gross uses 225 formation months and one-way turnover 219 eligible months. Free and Zero identify source characteristic intercepts; size is larger-firm positive.

Table IA.23: Signed training-reference exposure: current vs training (implementation)

Source intercept / no. of characteristics	Characteristic	Mean gross change (NAV units)	One-way turnover change (pp/month)
Free / 36	Book-to-market	0.031 [0.001,0.057]	1.24 [0.40,2.02]
Free / 36	Momentum	0.074 [-0.012,0.147]	4.44 [1.68,6.88]
Free / 36	Larger-firm size	-0.321 [-0.436,-0.211]	-6.92 [-9.65,-4.30]
Free / 153	Book-to-market	0.003 [-0.020,0.025]	0.52 [-0.15,1.14]
Free / 153	Momentum	0.075 [0.047,0.104]	3.41 [2.07,4.74]
Free / 153	Larger-firm size	-0.169 [-0.217,-0.124]	-4.58 [-6.17,-3.08]
Zero / 36	Book-to-market	0.018 [-0.009,0.042]	0.82 [0.03,1.53]
Zero / 36	Momentum	0.067 [-0.004,0.129]	4.35 [2.29,6.38]
Zero / 36	Larger-firm size	-0.301 [-0.423,-0.187]	-6.52 [-9.54,-3.76]
Zero / 153	Book-to-market	-0.000 [-0.023,0.023]	0.25 [-0.46,0.93]
Zero / 153	Momentum	0.070 [0.037,0.104]	2.96 [1.69,4.32]
Zero / 153	Larger-firm size	-0.222 [-0.269,-0.176]	-5.91 [-7.40,-4.45]

The policies preserve the Training rule's signed characteristic exposure, including negative targets. Entries are Current minus Training. Negative gross or turnover differences mean smaller positions or less trading under Current; they do not indicate a return gain. Gross changes are NAV multiples and one-way turnover changes are percentage points of NAV per month. All rules use the same formation stocks, annual loading coefficients and current cash constraint, but their current loading restrictions differ. Brackets contain pointwise 95% paired twelve-month circular calendar-block intervals (1,999 draws), conditional on frozen models and evaluated paths. Return quantities use 216 common months; gross uses 225 formation months and one-way turnover 219 eligible months. Free and Zero identify source characteristic intercepts; size is larger-firm positive.

Table IA.24: Unit gross: origin vs training (risk and mean return)

Source intercept / no. of characteristics	Characteristic	Variance reduction (%)	Mean excess return change (bp/month)
Free / 36	Book-to-market	16.83 [-4.62,36.44]	1.29 [-8.24,10.22]
Free / 36	Momentum	-12.95 [-40.57,11.23]	-2.31 [-11.79,7.28]
Free / 36	Larger-firm size	37.99 [27.62,47.22]	10.38 [-3.60,24.10]
Free / 153	Book-to-market	28.75 [1.32,45.35]	-3.13 [-17.54,7.68]
Free / 153	Momentum	-1.95 [-52.29,35.78]	-0.09 [-11.48,11.98]
Free / 153	Larger-firm size	29.40 [14.52,41.56]	7.62 [1.19,13.55]
Zero / 36	Book-to-market	17.28 [-4.35,36.85]	2.38 [-6.91,11.63]
Zero / 36	Momentum	-15.20 [-35.80,7.18]	4.24 [-5.00,13.83]
Zero / 36	Larger-firm size	41.65 [29.14,51.69]	6.42 [-6.25,19.18]
Zero / 153	Book-to-market	28.38 [4.16,44.50]	-4.73 [-18.17,5.88]
Zero / 153	Momentum	-9.34 [-49.48,27.26]	-0.34 [-11.93,11.35]
Zero / 153	Larger-firm size	29.09 [14.10,39.90]	10.36 [1.69,18.77]

The policies have equal unit gross, while their achieved characteristic exposures can differ. Variance reduction is relative to Training: positive values mean lower variance under Origin. Mean excess return and volatility differences are Origin minus Training, in monthly basis points. All rules use the same formation stocks, annual loading coefficients and current cash constraint, but their current loading restrictions differ. Brackets contain pointwise 95% paired twelve-month circular calendar-block intervals (1,999 draws), conditional on frozen models and evaluated paths. Return quantities use 216 common months; gross uses 225 formation months and one-way turnover 219 eligible months. Free and Zero identify source characteristic intercepts; size is larger-firm positive.

Table IA.25: Unit gross: origin vs training (implementation)

Source intercept / no. of characteristics	Characteristic	Mean gross change (NAV units)	One-way turnover change (pp/month)
Free / 36	Book-to-market	0.000 [-0.000,0.000]	0.87 [0.46,1.23]
Free / 36	Momentum	0.000 [-0.000,0.000]	0.83 [0.28,1.36]
Free / 36	Larger-firm size	0.000 [-0.000,0.000]	0.74 [-0.01,1.48]
Free / 153	Book-to-market	0.000 [-0.000,0.000]	0.53 [0.28,0.77]
Free / 153	Momentum	-0.000 [-0.000,0.000]	0.53 [-0.08,1.12]
Free / 153	Larger-firm size	-0.000 [-0.000,0.000]	-0.09 [-0.49,0.32]
Zero / 36	Book-to-market	0.000 [-0.000,0.000]	0.79 [0.39,1.16]
Zero / 36	Momentum	-0.000 [-0.000,0.000]	1.20 [0.30,2.20]
Zero / 36	Larger-firm size	-0.000 [-0.000,0.000]	0.30 [-0.42,1.05]
Zero / 153	Book-to-market	-0.000 [-0.000,0.000]	0.31 [-0.00,0.63]
Zero / 153	Momentum	-0.000 [-0.000,-0.000]	0.25 [-0.16,0.66]
Zero / 153	Larger-firm size	-0.000 [-0.000,0.000]	-0.24 [-0.70,0.26]

The policies have equal unit gross, while their achieved characteristic exposures can differ. Entries are Origin minus Training. Negative gross or turnover differences mean smaller positions or less trading under Origin; they do not indicate a return gain. Gross changes are NAV multiples and one-way turnover changes are percentage points of NAV per month. All rules use the same formation stocks, annual loading coefficients and current cash constraint, but their current loading restrictions differ. Brackets contain pointwise 95% paired twelve-month circular calendar-block intervals (1,999 draws), conditional on frozen models and evaluated paths. Return quantities use 216 common months; gross uses 225 formation months and one-way turnover 219 eligible months. Free and Zero identify source characteristic intercepts; size is larger-firm positive.

Table IA.26: Unit gross: current vs origin (risk and mean return)

Source intercept / no. of characteristics	Characteristic	Variance reduction (%)	Mean excess return change (bp/month)
Free / 36	Book-to-market	24.47 [15.05,32.24]	0.05 [-5.15,5.87]
Free / 36	Momentum	50.55 [29.26,64.67]	-0.43 [-8.40,7.37]
Free / 36	Larger-firm size	29.65 [13.60,43.40]	3.11 [-4.36,10.28]
Free / 153	Book-to-market	29.38 [15.71,39.14]	2.72 [-5.09,11.50]
Free / 153	Momentum	43.89 [24.68,56.46]	0.60 [-7.17,8.12]
Free / 153	Larger-firm size	23.52 [11.51,34.36]	1.57 [-2.62,6.25]
Zero / 36	Book-to-market	22.62 [14.46,30.30]	-1.04 [-6.50,4.59]
Zero / 36	Momentum	54.26 [33.67,64.95]	-0.39 [-9.25,8.46]
Zero / 36	Larger-firm size	31.35 [16.49,44.86]	5.11 [-2.45,13.04]
Zero / 153	Book-to-market	31.16 [18.35,41.47]	1.42 [-6.33,9.79]
Zero / 153	Momentum	43.65 [28.77,53.54]	2.44 [-6.23,10.88]
Zero / 153	Larger-firm size	25.70 [12.50,37.02]	3.02 [-2.12,8.54]

The policies have equal unit gross, while their achieved characteristic exposures can differ. Variance reduction is relative to Origin: positive values mean lower variance under Current. Mean excess return and volatility differences are Current minus Origin, in monthly basis points. All rules use the same formation stocks, annual loading coefficients and current cash constraint, but their current loading restrictions differ. Brackets contain pointwise 95% paired twelve-month circular calendar-block intervals (1,999 draws), conditional on frozen models and evaluated paths. Return quantities use 216 common months; gross uses 225 formation months and one-way turnover 219 eligible months. Free and Zero identify source characteristic intercepts; size is larger-firm positive.

Table IA.27: Unit gross: current vs origin (implementation)

Source intercept / no. of characteristics	Characteristic	Mean gross change (NAV units)	One-way turnover change (pp/month)
Free / 36	Book-to-market	-0.000 [-0.000,0.000]	-0.19 [-0.43,0.05]
Free / 36	Momentum	-0.000 [-0.000,0.000]	0.17 [-0.17,0.54]
Free / 36	Larger-firm size	-0.000 [-0.000,0.000]	-0.18 [-0.62,0.20]
Free / 153	Book-to-market	-0.000 [-0.000,0.000]	-0.09 [-0.28,0.08]
Free / 153	Momentum	0.000 [-0.000,0.000]	0.28 [-0.32,0.98]
Free / 153	Larger-firm size	0.000 [0.000,0.000]	-0.22 [-0.61,0.16]
Zero / 36	Book-to-market	-0.000 [-0.000,0.000]	-0.30 [-0.63,0.03]
Zero / 36	Momentum	-0.000 [-0.000,0.000]	0.10 [-0.31,0.51]
Zero / 36	Larger-firm size	-0.000 [-0.000,0.000]	0.05 [-0.32,0.40]
Zero / 153	Book-to-market	-0.000 [-0.000,0.000]	-0.09 [-0.31,0.09]
Zero / 153	Momentum	0.000 [-0.000,0.000]	0.40 [-0.07,0.95]
Zero / 153	Larger-firm size	0.000 [-0.000,0.000]	-0.28 [-0.67,0.10]

The policies have equal unit gross, while their achieved characteristic exposures can differ. Entries are Current minus Origin. Negative gross or turnover differences mean smaller positions or less trading under Current; they do not indicate a return gain. Gross changes are NAV multiples and one-way turnover changes are percentage points of NAV per month. All rules use the same formation stocks, annual loading coefficients and current cash constraint, but their current loading restrictions differ. Brackets contain pointwise 95% paired twelve-month circular calendar-block intervals (1,999 draws), conditional on frozen models and evaluated paths. Return quantities use 216 common months; gross uses 225 formation months and one-way turnover 219 eligible months. Free and Zero identify source characteristic intercepts; size is larger-firm positive.

Table IA.28: Unit gross: current vs training (risk and mean return)

Source intercept / no. of characteristics	Characteristic	Variance reduction (%)	Mean excess return change (bp/month)
Free / 36	Book-to-market	37.18 [19.60,52.05]	1.34 [-8.59,13.15]
Free / 36	Momentum	44.15 [25.00,62.61]	-2.74 [-10.22,4.90]
Free / 36	Larger-firm size	56.38 [44.91,64.48]	13.49 [-1.30,28.56]
Free / 153	Book-to-market	49.68 [19.69,65.30]	-0.41 [-12.44,12.25]
Free / 153	Momentum	42.79 [25.68,58.17]	0.51 [-11.35,13.05]
Free / 153	Larger-firm size	46.00 [28.77,60.03]	9.18 [2.37,15.71]
Zero / 36	Book-to-market	35.99 [20.09,50.06]	1.33 [-7.98,12.10]
Zero / 36	Momentum	47.31 [29.39,61.48]	3.85 [-4.46,12.24]
Zero / 36	Larger-firm size	59.94 [48.52,67.53]	11.53 [-4.15,27.60]
Zero / 153	Book-to-market	50.70 [24.10,66.30]	-3.31 [-14.20,8.25]
Zero / 153	Momentum	38.39 [20.28,55.93]	2.10 [-7.67,12.46]
Zero / 153	Larger-firm size	47.31 [30.27,59.89]	13.38 [4.20,21.94]

The policies have equal unit gross, while their achieved characteristic exposures can differ. Variance reduction is relative to Training: positive values mean lower variance under Current. Mean excess return and volatility differences are Current minus Training, in monthly basis points. All rules use the same formation stocks, annual loading coefficients and current cash constraint, but their current loading restrictions differ. Brackets contain pointwise 95% paired twelve-month circular calendar-block intervals (1,999 draws), conditional on frozen models and evaluated paths. Return quantities use 216 common months; gross uses 225 formation months and one-way turnover 219 eligible months. Free and Zero identify source characteristic intercepts; size is larger-firm positive.

Table IA.29: Unit gross: current vs training (implementation)

Source intercept / no. of characteristics	Characteristic	Mean gross change (NAV units)	One-way turnover change (pp/month)
Free / 36	Book-to-market	-0.000 [-0.000,0.000]	0.67 [0.32,1.00]
Free / 36	Momentum	-0.000 [-0.000,0.000]	1.00 [0.59,1.45]
Free / 36	Larger-firm size	-0.000 [-0.000,0.000]	0.56 [-0.09,1.31]
Free / 153	Book-to-market	-0.000 [-0.000,0.000]	0.44 [0.20,0.69]
Free / 153	Momentum	-0.000 [-0.000,0.000]	0.81 [0.34,1.27]
Free / 153	Larger-firm size	0.000 [-0.000,0.000]	-0.31 [-0.68,0.05]
Zero / 36	Book-to-market	0.000 [-0.000,0.000]	0.49 [0.12,0.84]
Zero / 36	Momentum	-0.000 [-0.000,0.000]	1.30 [0.56,2.12]
Zero / 36	Larger-firm size	-0.000 [-0.000,0.000]	0.35 [-0.46,1.24]
Zero / 153	Book-to-market	-0.000 [-0.000,0.000]	0.22 [-0.09,0.53]
Zero / 153	Momentum	-0.000 [-0.000,0.000]	0.65 [0.29,1.08]
Zero / 153	Larger-firm size	0.000 [-0.000,0.000]	-0.52 [-0.87,-0.17]

The policies have equal unit gross, while their achieved characteristic exposures can differ. Entries are Current minus Training. Negative gross or turnover differences mean smaller positions or less trading under Current; they do not indicate a return gain. Gross changes are NAV multiples and one-way turnover changes are percentage points of NAV per month. All rules use the same formation stocks, annual loading coefficients and current cash constraint, but their current loading restrictions differ. Brackets contain pointwise 95% paired twelve-month circular calendar-block intervals (1,999 draws), conditional on frozen models and evaluated paths. Return quantities use 216 common months; gross uses 225 formation months and one-way turnover 219 eligible months. Free and Zero identify source characteristic intercepts; size is larger-firm positive.

Table IA.30: When the Training rule reverses the characteristic direction

Source intercept / no. of characteristics	Characteristic	Negative target months	Formation months	Near-zero target months
Free / 36	Book-to-market	0	225	0
Free / 36	Momentum	3	225	0
Free / 36	Larger-firm size	10	225	0
Free / 153	Book-to-market	0	225	0
Free / 153	Momentum	0	225	0
Free / 153	Larger-firm size	0	225	0
Zero / 36	Book-to-market	0	225	0
Zero / 36	Momentum	1	225	0
Zero / 36	Larger-firm size	15	225	0
Zero / 153	Book-to-market	0	225	0
Zero / 153	Momentum	0	225	0
Zero / 153	Larger-firm size	0	225	0

Counts describe the characteristic exposure of the Training unit-gross portfolio, before matching the other references to that signed target. A negative target points against the original positive characteristic mandate and remains negative in all three matched portfolios. Near-zero refers to the fixed absolute numerical threshold of 10^{-12} . All 225 formation months remain in the holdings comparisons. Free and Zero identify source characteristic intercepts; size denotes the larger-firm-positive profile.

IA.6 Current and Annual-Origin Portfolios at the Original Positive Target

The positive-target comparison holds fixed the exposure $\xi_t^0 = (w_t^0)'Z_t a > 0$ of the original cash-neutral, unit-gross characteristic portfolio. Let v_t^M denote the unit-gross holdings produced by the cash-constrained projection in Appendix IA.1.7 with reference M , and let $\eta_t^M = (v_t^M)'Z_t a$. This inner product is the signed characteristic exposure per unit gross under reference M . Its sign depends on the selected direction; the positive norm ratio η_t in the Current projection identity follows from that projection's orthogonality. The matched holdings are $w_t^M = (\xi_t^0 / \eta_t^M) v_t^M$ whenever $\eta_t^M \neq 0$. Their gross holdings are $\xi_t^0 / |\eta_t^M|$. This scaling preserves the original economic exposure and cash neutrality while allowing the required stock positions to differ. It does not impose common current loading neutrality across the two references.

All 5,400 current and origin formation requests have positive residual characteristic exposure and satisfy their target and cash constraints. They cover twelve model–characteristic combinations, two references, and 225 formation months. Current holdings also satisfy the current stock-loading restriction; origin holdings generally do not between annual updates. The rules agree at March formation because their reference matrices then agree. Actual-stock reconstruction checks the holdings, exposures, and subsequent payoffs against the archived rules. Table IA.33 reports the remaining current-loading projection and the observation counts for every policy.

Table IA.31 reports the complete twelve-member risk family. The statistic measures the percentage of origin variance removed, using the same 216 return observations for every comparison. The inference resamples twelve-month circular blocks on the full 225-month formation calendar, carrying its missing-value indicators with the sampled positions. Studentized maximum absolute statistics from 19,999 common draws provide simultaneous intervals within this family. The family was fixed as an exploratory follow-up after the three-reference positive-target feasibility exercise encountered training-rule wealth failures. It conditions on the estimated model paths and uses a distinct inference family from both the signed-target and library-expansion comparisons. Appendix IA.8 gives the studentization and calendar conventions.

Table IA.32 separates return risk from the payoff and trading consequences of the policies. Mean excess return declines from origin to current for unrestricted 36-characteristic momentum (25.30 to 18.30 basis points per month), zero-intercept 36-characteristic book-to-market (7.87 to 6.23), and zero-intercept 36-characteristic momentum (−15.58 to −24.74). Unrestricted 153-characteristic momentum has slightly higher average one-way turnover under current measurement, 57.84% rather than 57.54% of NAV. These levels are descriptive and are not covered by

Table IA.31: Current versus Origin: variance at the original positive target

Source intercept / no. of characteristics	Characteristic	Variance reduction vs. Origin (%)	Familywise 95% CI (%)	Adjusted p
Free / 36	Book-to-market	28.34	[5.28, 51.40]	0.01460
Free / 36	Momentum	57.22	[10.96, 103.49]	0.01395
Free / 36	Larger-firm size	55.81	[11.68, 99.93]	0.01205
Free / 153	Book-to-market	35.74	[-1.64, 73.11]	0.06255
Free / 153	Momentum	40.29	[9.78, 70.81]	0.00920
Free / 153	Larger-firm size	32.17	[-1.58, 65.91]	0.06345
Zero / 36	Book-to-market	26.62	[6.42, 46.83]	0.00920
Zero / 36	Momentum	69.39	[11.91, 126.87]	0.01600
Zero / 36	Larger-firm size	57.94	[9.79, 106.09]	0.01635
Zero / 153	Book-to-market	35.67	[2.33, 69.01]	0.03450
Zero / 153	Momentum	41.69	[15.72, 67.66]	0.00250
Zero / 153	Larger-firm size	36.48	[-2.68, 75.64]	0.07125

Each row compares Current and Origin portfolios at the same original positive characteristic target and current cash neutrality. The estimate is $100[1 - \text{Var}(r^{\text{current}}) / \text{Var}(r^{\text{origin}})]$, in percent of origin variance; positive values favor Current. Nine of twelve simultaneous intervals exclude zero. Brackets give familywise 95% intervals within these twelve comparisons; p -values are adjusted using the same maximum absolute statistic and 19,999 common calendar-block draws. Returns use the original 216 common months. The unrestricted and zero-intercept source models each use 36 or 153 characteristics, FFC6, and three latent factors. Size means a positive larger-firm exposure. All endpoints are retained, including three upper endpoints above 100% arising from the untransformed symmetric approximation; these are not feasible reductions exceeding the entire reference variance. Current and Origin impose different current loading restrictions. This separate family was specified after the training-reference feasibility outcome and conditions on the fitted models.

the variance family's simultaneous intervals. They allow the investor to assess a measured risk reduction alongside the position and trading requirements needed to obtain it.

The original calendar is retained throughout. Across the 5,400 requests there are 120 unavailable one-period wealth observations, 60 for each reference. No nonpositive wealth factor is observed on the remaining records, but the missing observations prevent a claim of uninterrupted investment survival over all 225 months. The common return mask also excludes dates needed to keep comparisons aligned, so it is distinct from policy-specific wealth availability. Maximum formation gross is 12.3804 times NAV for origin and 4.0945 for current. These observed maxima describe the unconstrained policies; they are not leverage limits or evidence about financing availability.

Table IA.32: Current and Origin: payoffs and position requirements at the same positive target

Source intercept / no. of characteristics	Characteristic	Rule	Mean excess return (bp/month)	Volatility (bp/month)	Median gross/NAV (multiple)	Maximum gross/NAV (multiple)	One-way turnover fraction of NAV per month
Free / 36	Book-to-market	Origin	18.14	136.19	1.460	1.972	0.26830
		Current	18.32	115.29	1.437	1.733	0.25887
Free / 36	Momentum	Origin	25.30	312.30	2.244	8.861	0.87293
		Current	18.30	204.26	2.306	3.031	0.81404
Free / 36	Larger-firm size	Origin	-15.26	414.13	3.554	8.884	1.12778
		Current	12.85	275.30	3.334	3.958	0.92989
Free / 153	Book-to-market	Origin	12.34	140.30	1.319	1.839	0.25254
		Current	17.03	112.47	1.314	1.504	0.24562
Free / 153	Momentum	Origin	32.76	195.75	1.579	2.742	0.57536
		Current	34.42	151.25	1.587	2.242	0.57838
Free / 153	Larger-firm size	Origin	-20.61	184.70	1.929	3.276	0.54506
		Current	-15.66	152.12	1.921	2.286	0.51285
Zero / 36	Book-to-market	Origin	7.87	134.36	1.476	1.905	0.28793
		Current	6.23	115.09	1.450	1.764	0.27623
Zero / 36	Momentum	Origin	-15.58	385.57	2.314	12.380	0.92294
		Current	-24.74	213.32	2.394	3.137	0.83174
Zero / 36	Larger-firm size	Origin	-0.29	421.43	3.447	11.679	1.08863
		Current	31.60	273.32	3.404	4.094	0.91687
Zero / 153	Book-to-market	Origin	1.85	136.20	1.325	1.730	0.24820
		Current	4.79	109.24	1.309	1.486	0.24264
Zero / 153	Momentum	Origin	-5.11	207.27	1.693	3.224	0.59689
		Current	-1.60	158.28	1.702	2.300	0.59383
Zero / 153	Larger-firm size	Origin	-18.51	191.64	1.931	3.454	0.52874
		Current	-11.00	152.74	1.919	2.260	0.49300

Each pair preserves the same original positive characteristic target, current cash neutrality, annual model coefficients, and formation stocks. The paired rows show how return risk, average excess returns, gross holdings and trading change together; the lower-risk rule does not uniformly improve the other outcomes. Means and volatilities are monthly basis points over 216 common payoff months. Gross/NAV quantiles and maxima use 225 formation months. One-way turnover is the mean fraction of NAV traded per month over 219 eligible months; the displayed turnover values are fractions, not percentages. Origin generally retains some current loading exposure; Current is current-loading neutral. Free and Zero denote the source characteristic intercept, and size is a larger-firm target. All entries are descriptive levels, with no familywise inference for means, gross, or turnover.

Table IA.33: Current-loading alignment and available wealth observations at the positive target

Source intercept / no. of characteristics	Characteristic	Rule	95th percentile gross/NAV (multiple)	Median loading alignment (L^2 norm ratio)	Maximum loading alignment (L^2 norm ratio)	Missing wealth observations (months)
Free / 36	Book-to-market	Origin	1.699	0.1265	0.2818	5
		Current	1.612	5.15×10^{-15}	1.75×10^{-14}	5
Free / 36	Momentum	Origin	3.740	0.2183	0.6641	5
		Current	2.665	8.44×10^{-15}	2.94×10^{-14}	5
Free / 36	Larger-firm size	Origin	5.801	0.1802	0.5765	5
		Current	3.843	1.59×10^{-14}	6.55×10^{-14}	5
Free / 153	Book-to-market	Origin	1.536	0.1202	0.3384	5
		Current	1.455	6.05×10^{-15}	1.58×10^{-14}	5
Free / 153	Momentum	Origin	1.962	0.2412	0.6733	5
		Current	1.822	7.92×10^{-15}	1.82×10^{-14}	5
Free / 153	Larger-firm size	Origin	2.645	0.1574	0.4002	5
		Current	2.199	1.32×10^{-14}	2.55×10^{-14}	5
Zero / 36	Book-to-market	Origin	1.684	0.1262	0.2672	5
		Current	1.608	5.16×10^{-15}	1.49×10^{-14}	5
Zero / 36	Momentum	Origin	3.875	0.2261	0.6523	5
		Current	2.818	1.18×10^{-14}	4.44×10^{-14}	5
Zero / 36	Larger-firm size	Origin	5.397	0.1726	0.5406	5
		Current	3.849	1.69×10^{-14}	5.70×10^{-14}	5
Zero / 153	Book-to-market	Origin	1.498	0.1196	0.3014	5
		Current	1.447	5.58×10^{-15}	1.41×10^{-14}	5
Zero / 153	Momentum	Origin	2.227	0.2509	0.6843	5
		Current	1.898	8.92×10^{-15}	1.86×10^{-14}	5
Zero / 153	Larger-firm size	Origin	2.587	0.1581	0.3970	5
		Current	2.179	1.22×10^{-14}	3.50×10^{-14}	5

The Current and Origin policies preserve the same original positive characteristic target but leave different exposure to current stock loadings. Current-loading alignment is $\|P_{H_t} w_t\|_2 / \|w_t\|_2$, where P_{H_t} projects stock weights onto the current cash-inclusive loading space $H_t = Z_t[e_0, C, B]$. This stock-weight alignment ratio is unchanged by rescaling the entire portfolio; it is not a return-variance share. Current alignment is numerically zero, while Origin generally retains current loading exposure. The gross column is the 95th percentile in multiples of NAV. Entries summarize all 225 formation requests for each policy. Free and Zero refer to the source characteristic intercept; size is larger-firm positive. Missing wealth counts unavailable subsequent one-period wealth observations; none is filled or treated as positive. Every policy has five such observations, totaling 120 across the 24 policies. These are descriptive exposure and availability checks, not evidence of uninterrupted investment survival.

IA.7 Exposure Cancellation and the Limits of Unconstrained Scaling

The three reference rules can all produce finite holdings at the original positive target while requiring very different gross positions. Table IA.34 isolates June 2008 for the zero-intercept 36-characteristic larger-firm-size mandate. All three portfolios use the same 2,441 stocks and positive target, $\xi_t^0 = 0.335497911$. For the training reference, the unit-gross positive and negative characteristic contributions are approximately 0.1264627 and -0.1264237 . They nearly cancel, leaving $\eta_t^{\text{train}} = 0.0000389761$. Since matched gross equals $\xi_t^0 / |\eta_t^{\text{train}}|$, the selected direction requires 8,607.78 times NAV. The denominator is positive: a sign reversal is not necessary for this amplification.

Table IA.34: The same positive size target under three exposure references: June 2008

Quantity	Training Q	Origin Q	Current Q
Positive characteristic target	0.33549791	0.33549791	0.33549791
Unit-gross remaining exposure	0.00003898	0.08413278	0.08505412
Gross holdings / NAV	8607.78	3.99	3.94
Current loading alignment (L^2 norm ratio)	0.508481	0.274707	2.52×10^{-14}
Reference Gram condition number	289.95	357.89	374.88
Next-period model wealth factor	-90.207438	0.973842	1.008599

The rules use the same positive larger-firm-size target 0.335497911 on 2,441 formation stocks, under a zero-intercept source model with 36 characteristics, FFC6 and three latent factors. Each satisfies current cash neutrality. Unit-gross residual exposure is the signed characteristic exposure before rescaling; dividing the target by its absolute value gives the required gross in multiples of NAV. The Training direction leaves almost no net target exposure and therefore requires the largest position. Current-loading alignment is $\|P_{H_t} w_t\|_2 / \|w_t\|_2$ on the current cash-inclusive stock-loading space and is not a variance share. The rules have different current loading restrictions and do not solve a common minimum-gross problem. The following-month wealth factor is model wealth relative to the NAV used at this formation under the risk-free cash convention, without security-specific financing charges or margin liquidation.

Its negative training value is a first model-path termination, not an observed fund default.

The origin and current rules retain unit-gross exposure of approximately 0.08413 and 0.08505, requiring gross of 3.987719 and 3.944523 times NAV. Their stock weights also differ in alignment with current loadings. The proportion of the weight norm projected onto the current loading space, $\|P_{H_t} w_t\|_2 / \|w_t\|_2$, is 0.508481 for training, 0.274707 for origin, and numerically zero for current. Thus the references retain the same characteristic target and cash constraint but impose different current loading restrictions. The alternative directions demonstrate that 8,607.78 is not a universal capital requirement for this target. None of these calculations solves a minimum-gross problem over all admissible portfolios.

Near-cancellation of positive and negative characteristic contributions produces the large position. The selected training-reference projected Gram matrix has condition number 289.95, and its stored linear-system residual is approximately 2.22×10^{-16} . Independent stock-level reconstruction and QR/SVD comparisons reproduce the weights and wealth calculation. The top ten absolute stock weights account for 1.69% of gross, so the extreme gross is not concentrated in a few names as a fraction of the position. It is the scaling of the entire nearly canceling direction that creates the exceptionally large dollar holdings. The exact overlap–position-shape identity in Section V applies to the current orthogonal projection; it is not imposed on this historical-reference direction.

Table IA.35 records the first nonpositive model-wealth event in each affected training strategy. The two size cases arise from June 2008 formation, under unrestricted and zero intercepts; the momentum case arises from November 2008 formation under an unrestricted intercept. All three use 36 characteristics. The zero-intercept size portfolio’s following-month wealth factor is -90.2074 , relative to the NAV at that formation. These are outcomes of the uncapped model policies, not observations of real fund defaults. Once a path reaches its first nonpositive wealth factor, later formation requests remain useful for diagnosing the rule but cannot be interpreted as a continuation of that investment path.

Table IA.35: First nonpositive model-wealth events under the Training reference

Source intercept / no. of characteristics	Characteristic	Formation (YYYYMM)	Return (YYYYMM)	Gross/NAV	Model wealth factor
Free / 36	Larger-firm size	200806	200807	150.38	-0.888021
Zero / 36	Larger-firm size	200806	200807	8607.78	-90.207438
Free / 36	Momentum	200811	200812	69.42	-1.388247

Each row identifies the first nonpositive one-period model-wealth factor for a Training strategy maintaining its original positive characteristic target. All rows use 36-characteristic source models; Free and Zero identify the source characteristic intercept and size is larger-firm positive. Formation month determines the holdings; return month is the next monthly payoff. Gross is the sum of absolute stock weights per unit of formation NAV. Wealth factor includes the risk-free cash financing convention and the realized excess stock payoff, but no margin liquidation or security-specific financing charges. These are model-path terminations, not observed fund defaults. Only the first nonpositive event of each model–characteristic strategy is listed. Later requests on the terminated path are counterfactual diagnostics.

Of 61 training-reference formation requests with gross above ten times NAV, 24 precede a first termination, three are the first terminal requests, and 34 occur after termination. All 61 belong to the 36-characteristic library; 54 occur during the March 2007–December 2009 portion of the

formation sample. Table IA.36 reports the distribution for all 36 policies, including later counterfactual requests. Across all 8,100 requests, 180 one-period wealth observations are unavailable, 60 under each reference. The ledger preserves these observations and all first-terminal and postterminal states. Formation-time solvability therefore documents what a rule asks an investor to hold; it does not by itself establish a continuously financeable strategy.

Table IA.36: Gross holdings across all positive-target formation requests

No. of char.	Characteristic	Reference	Median	95th pct.	Maximum	Gross/NAV above 10	First terminal request	Postterminal request
			Gross holdings / NAV			(requests)	(requests)	(requests)
<i>Free source characteristic intercept</i>								
36	Book-to-market	Training	1.387	1.697	2.13	0	0	0
36	Book-to-market	Origin	1.460	1.699	1.97	0	0	0
36	Book-to-market	Current	1.437	1.612	1.73	0	0	0
36	Momentum	Training	2.020	3.910	476.57	5	1	204
36	Momentum	Origin	2.244	3.740	8.86	0	0	0
36	Momentum	Current	2.306	2.665	3.03	0	0	0
36	Larger-firm size	Training	4.705	24.220	150.38	25	1	209
36	Larger-firm size	Origin	3.554	5.801	8.88	0	0	0
36	Larger-firm size	Current	3.334	3.843	3.96	0	0	0
153	Book-to-market	Training	1.295	1.551	1.98	0	0	0
153	Book-to-market	Origin	1.319	1.536	1.84	0	0	0
153	Book-to-market	Current	1.314	1.455	1.50	0	0	0
153	Momentum	Training	1.468	1.805	2.58	0	0	0
153	Momentum	Origin	1.579	1.962	2.74	0	0	0
153	Momentum	Current	1.587	1.822	2.24	0	0	0
153	Larger-firm size	Training	2.290	3.626	5.50	0	0	0
153	Larger-firm size	Origin	1.929	2.645	3.28	0	0	0
153	Larger-firm size	Current	1.921	2.199	2.29	0	0	0
<i>Zero source characteristic intercept</i>								
36	Book-to-market	Training	1.409	1.691	2.05	0	0	0
36	Book-to-market	Origin	1.476	1.684	1.90	0	0	0

Continued on the next page

Table IA.36 continued

No. of char.	Characteristic	Reference	Median	95th pct.	Maximum	Gross/NAV above 10	First terminal request	Postterminal request
			Gross holdings / NAV			(requests)	(requests)	(requests)
36	Book-to-market	Current	1.450	1.608	1.76	0	0	0
36	Momentum	Training	2.142	3.987	377.76	4	0	0
36	Momentum	Origin	2.314	3.875	12.38	2	0	0
36	Momentum	Current	2.394	2.818	3.14	0	0	0
36	Larger-firm size	Training	4.478	20.700	8607.78	27	1	209
36	Larger-firm size	Origin	3.447	5.397	11.68	1	0	0
36	Larger-firm size	Current	3.404	3.849	4.09	0	0	0
153	Book-to-market	Training	1.297	1.575	1.85	0	0	0
153	Book-to-market	Origin	1.325	1.498	1.73	0	0	0
153	Book-to-market	Current	1.309	1.447	1.49	0	0	0
153	Momentum	Training	1.546	1.902	2.63	0	0	0
153	Momentum	Origin	1.693	2.227	3.22	0	0	0
153	Momentum	Current	1.702	1.898	2.30	0	0	0
153	Larger-firm size	Training	2.379	3.825	6.40	0	0	0
153	Larger-firm size	Origin	1.931	2.587	3.45	0	0	0
153	Larger-firm size	Current	1.919	2.179	2.26	0	0	0

Table IA.36 uses all 225 formation requests per policy. The $G > 10$ column counts requests above ten times NAV. The first-terminal and postterminal columns count states among all requests, not just the $G > 10$ subset; together the training rules have three first-terminal and 622 postterminal requests. Gross quantiles and maxima include the postterminal counterfactual requests. No trimming or leverage cap is imposed.

IA.8 Uncertainty and Additional Performance Measures

The uncertainty calculations resample common calendar blocks across all compared models and targets. Each draw samples blocks of twelve consecutive formation months, wraps at the calendar boundary, and keeps the original missing-value indicators. The original pointwise calculations use 1,999 draws. Their intervals use the 2.5th and 97.5th percentiles of paired differences. The same sampled calendar positions apply to both portfolios in each comparison, so common market shocks are preserved in the contrast.

These intervals condition on the estimated source models and the evaluated payoff paths. They quantify the sampling variability of those realized comparisons, rather than rerunning characteristic selection and factor estimation in each draw. We report all four main source specifications and the fixed economic anchors. The state and holdings diagnostic follow-ups were conducted after the initial model comparisons; the replication documentation retains their chronology. They are not presented as results from an untouched validation period.

IA.8.1 Separate Simultaneous Inference for the Portfolio Comparisons

The library-expansion family contains twelve volatility differences: two source-intercept specifications, three characteristics, and two portfolio rules. The signed-reference family contains twelve current-versus-origin variance reductions: two intercept specifications, two library sizes, and three characteristics. The original-positive-target family has the same model–characteristic roster as the signed family but a different exposure target. The three sets receive separate simultaneous intervals. No joint coverage over their thirty-six comparisons is asserted. The complete rows, including all intervals that contain zero, are reported in the corresponding tables above.

Each family uses 19,999 shared draws of twelve-month circular calendar blocks. A draw samples nineteen block starts uniformly from the 225 formation positions, concatenates the blocks, and truncates at 225. The original outcome mask travels with the sampled positions, so the nine excluded formation dates are not removed before resampling. The observed estimators use the 216 common returns and sample variances with denominator 215; a resample uses its own number of observed returns. The PCG64 seeds are 710204 for the library and signed-reference comparisons and 710205 for the positive-target comparison. These are follow-up inference protocols conditional on the frozen fitted models. The positive-target family was approved after the earlier three-reference feasibility check; it does not reinstate the failed training-reference wealth gate or assume returns in unobserved months.

For clarity, write $T_c = 225$ for the number of calendar positions, I_t for the common return indicator, and $n = \sum_t I_t$ for the number observed in a given sample. Here n counts observed return months, whereas n_t in the stock-formation formulas counts stocks. For a portfolio return series, let $\bar{r}$ be its observed sample mean, v its sample variance, and $c_n = n/(n-1)$. The leading-order variance influence is

$$\psi_{v,t} = \frac{T_c}{n} c_n I_t \left\{ (r_t - \bar{r})^2 - \frac{v}{c_n} \right\}, \quad (\text{IA.8.1})$$

with zero assigned to unobserved positions. The finite-sample correction c_n is held fixed in the derivative within each sample and recomputed in each draw. The scaling by T_c/n retains the calendar spacing when forming the long-run variance.

For variance reduction $\theta = 100(1 - v_C/v_O)$, where C and O denote current and origin rules, the influence is $\psi_{\theta,t} = -100\psi_{v_C,t}/v_O + 100v_C\psi_{v_O,t}/v_O^2$. For the library volatility difference $d = 10^4(\sqrt{v_{153}} - \sqrt{v_{36}})$ in monthly basis points, it is $\psi_{d,t} = 10^4\{\psi_{v_{153},t}/(2\sqrt{v_{153}}) - \psi_{v_{36},t}/(2\sqrt{v_{36}})\}$. The scalar θ is a risk comparison, distinct from the vector of projection coefficients in the portfolio solution; subscripts C and O identify portfolio rules here. We center each resulting influence sequence over all T_c calendar positions. Its standard error is

$$\widehat{\text{se}} = \left[\frac{\widehat{\gamma}_0 + 2\sum_{h=1}^{11}(1-h/12)\widehat{\gamma}_h}{T_c} \right]^{1/2}, \quad \widehat{\gamma}_h = T_c^{-1} \sum_{t=h+1}^{T_c} \widetilde{\psi}_t \widetilde{\psi}_{t-h}, \quad (\text{IA.8.2})$$

where $\widetilde{\psi}$ is the centered influence. These autocovariances are noncircular, although the bootstrap blocks wrap around the original calendar. The same formulas are recomputed within every draw, using its own means, variances, and observed count.

Let $\widehat{\theta}_j$ denote the estimated contrast for member j of a family and $\widehat{\theta}_{bj}^*$ its value in draw b . We form

$$M_b = \max_{1 \leq j \leq 12} \left| \frac{\widehat{\theta}_{bj}^* - \widehat{\theta}_j}{\widehat{\text{se}}_{bj}^*} \right|. \quad (\text{IA.8.3})$$

The critical value is the 19,000th ascending order statistic of the 19,999 values of M_b . The simultaneous interval is $\widehat{\theta}_j \pm c_{.95}\widehat{\text{se}}_j$, and the adjusted p -value is $(1 + \#\{b : M_b \geq |\widehat{\theta}_j/\widehat{\text{se}}_j|\})/20,000$. All draws are retained; the computations encountered no invalid denominators or standard errors. These intervals use a conditional, weak-dependence approximation rather than incorporating factor-model reestimation. Symmetric variance-reduction intervals are reported without truncation when their upper endpoints exceed 100%; those endpoints are features of the approximation, not attainable reductions in variance.

A separate implementation reproduces the index arrays, all draw estimates and studentizers, and every reported interval and adjusted p -value. For the positive-target comparison, the return inputs were also independently rebuilt from the original stocks and frozen annual models. The checks verify the numerical implementation of the stated experiment. They do not change its exposure target, observed return calendar, or conditional inferential scope.

Table IA.37: Expanding the library: changes in average-return pricing errors

Source intercept	Measure	153 minus 36 (bp/month)	Lower 95% endpoint	Upper 95% endpoint
free	Forecast mean error RMS	-2.215	-5.056	2.078
free	Realized factor pricing RMS	-0.929	-2.318	1.338
zero	Forecast mean error RMS	-3.902	-6.336	1.792
zero	Realized factor pricing RMS	-2.055	-5.851	1.123

Each entry is the pricing error under 153 characteristics minus the corresponding error under 36, on identical test portfolios and dates. Negative changes favor the larger library. All changes and interval endpoints are monthly basis points. The factor-only specification and target-regression intercept are kept fixed within a comparison. Brackets give paired, pointwise 95% conditional calendar-block intervals, not the simultaneous portfolio-volatility family.

Table IA.38 reports time-series and monthly cross-section R^2 alongside both forecast benchmarks. All the measures use the common target portfolios. Their different aggregation rules emphasize different dimensions of model fit: pooled total R^2 gives more influence to large squared payoffs, average time-series R^2 weights each balanced target equally, and average monthly cross-section R^2 weights each scored month equally.

Table IA.38: Return fit and forecasts under alternative aggregations and benchmarks

Factor-payoff model	Target-regression intercept	Time-series R^2 (%)	Cross-section R^2 (%)	Predictive R^2 (%) zero benchmark	Predictive R^2 (%) past-mean benchmark
Unrestricted, 36	Excluded	61.07	48.32	0.02	-0.54
Unrestricted, 36	Included	61.85	48.93	0.50	-0.06
FFC6	Excluded	38.28	13.82	0.16	-0.40
FFC6	Included	37.81	13.10	0.20	-0.36
Unrestricted, 153	Excluded	58.42	43.81	0.32	-0.25
Unrestricted, 153	Included	58.92	44.45	0.47	-0.09
Zero intercept, 36	Excluded	60.16	46.42	-0.34	-0.91
Zero intercept, 36	Included	61.47	47.74	0.51	-0.05
Zero intercept, 153	Excluded	58.38	43.25	0.22	-0.35
Zero intercept, 153	Included	59.01	44.73	0.48	-0.08

The columns distinguish reconstruction of realized target returns from forecasts using past information. Entries are percentages: higher fit measures indicate smaller squared errors under the stated aggregation, and higher predictive measures indicate smaller errors relative to the named forecast benchmark. Intercept indicates the target-regression intercept; source intercept specifications are named in the model column. All forecast coefficients and factor means use past information. Pred. zero and Pred. past mean differ only in the benchmark. The time-series and monthly cross-section columns weight targets and months differently; all use the common target portfolios.

IA.9 All Economic Targets and Holdings

The target-by-target hedge comparison below retains all 36 evaluation portfolios. Entries compare FFC6 plus latent factors with FFC6 plus the three fixed economic portfolios. Positive values indicate a larger reduction in that target's time-series variance from using the latent factors. The code retains each model's levels as well as these differences, so the comparison can be reconstructed without selecting targets according to the result.

Table IA.39: Hedging differences for every test portfolio

Entries are percentage-point differences in each target's hedge variance reduction: FFC6 plus three latent portfolios minus FFC6 plus book-to-market, momentum, and larger-firm-size portfolios. Positive values favor the latent menu. Each target's own unhedged variance is the denominator. Slopes use past returns and outcomes use 216 common subsequent months. Free/Zero identify source characteristic intercepts; 36/153 count characteristics. Target codes are defined in Table IA.51. These are descriptive differences for individual targets.

Target	Free / 36 (pp)	Free / 153 (pp)	Zero / 36 (pp)	Zero / 153 (pp)
market_equity	-3.69	-8.01	-3.58	-7.66
be_me	-8.49	-10.86	-9.77	-11.24
at_me	-3.42	-6.01	-4.97	-6.56
sale_me	4.11	4.02	3.53	4.36
ebitda_mev	0.69	-3.12	0.63	-2.71
gp_at	1.04	2.53	2.02	4.03
ope_be	0.74	-2.88	0.30	-2.88
ni_be	3.29	-0.03	3.04	0.56
ebit_bev	2.40	-1.45	2.23	-0.80
cop_at	-7.64	-9.85	-6.90	-12.79
at_gr1	-0.32	-5.13	-0.08	-2.92
capx_gr1	1.95	-0.28	0.88	-1.35
inv_gr1a	2.07	-2.10	0.59	-2.67
noa_gr1a	-0.77	-1.54	-0.80	0.77
ret_12_1	-5.94	-6.87	-5.97	-7.11
ret_6_1	-0.35	-3.32	-2.34	-4.58
ret_12_7	-4.48	-5.13	-3.42	-4.26
resff3_12_1	-10.09	-9.82	-11.92	-10.65
ret_1_0	21.24	3.60	13.79	2.97
beta_60m	4.24	2.37	3.52	2.22
ivol_capm_252d	3.14	0.59	3.33	1.48
rvol_21d	3.04	0.08	2.74	0.72
betabab_1260d	4.62	1.32	2.12	-2.06
qmj	1.60	-1.54	1.60	-0.16
f_score	1.02	-1.93	0.26	-2.49
o_score	1.19	-1.79	1.12	-1.40

Target	Free / 36 (pp)	Free / 153 (pp)	Zero / 36 (pp)	Zero / 153 (pp)
z_score	2.71	1.02	3.22	0.07
niq_be	1.97	-1.36	1.78	-0.53
oaccruals_at	7.30	4.26	7.12	4.46
taccruals_at	4.43	0.88	3.44	1.21
debt_me	1.71	1.47	0.81	1.40
netdebt_me	7.23	9.60	7.69	10.40
ami_126d	-2.98	-9.99	-2.85	-8.05
dolvol_126d	-0.71	-8.02	-0.12	-7.12
turnover_126d	8.49	5.93	8.92	6.65
seas_1_1na	-5.25	-7.92	-6.62	-8.50

The following holdings table reports every characteristic, source-library, and source-intercept combination. Mean returns and volatility are in monthly basis points; SR is the mean excess return divided by its standard deviation, multiplied by $\sqrt{12}$. Turnover is one-way monthly turnover in percent. “Unhedged” is the demeaned characteristic-managed source-stock portfolio, normalized to unit gross holdings. These portfolios use the source securities and subsequent returns, while the pricing and factor-hedge tests in the main text use the separate target securities.

Table IA.40: Returns, risk, and trading of all stock portfolios

All policies hold source stocks with cash neutrality and unit gross. Coefficient imposes coefficient orthogonality; Stock exposure imposes neutrality to current estimated stock loadings; Unhedged holds the demeaned characteristic profile. The desired characteristic exposure can differ across these rules. Mean excess returns and volatility use 216 common monthly returns; annualized Sharpe ratios equal the mean divided by volatility times $\sqrt{12}$. One-way turnover is percent of NAV per month over 219 eligible months. Free/Zero denote source characteristic intercepts and 36/153 count characteristics. Size is larger-firm positive.

Source intercept / no. of char.	Target	Rule	Mean excess return (bp/month)	Volatility (bp/month)	Annualized Sharpe ratio	One-way turnover (% NAV/month)
Free153	Book-to-market	Coefficient	24.49	181.03	0.47	13.05
Free153	Book-to-market	Stock exposure	12.93	83.09	0.54	18.40
Free153	Book-to-market	Unhedged	18.20	191.98	0.33	10.92
Free153	Firm size	Coefficient	9.23	205.45	0.16	9.87
Free153	Firm size	Stock exposure	-8.54	79.99	-0.37	26.52

Source intercept / no. of char.	Target	Rule	Mean excess return (bp/month)	Volatility (bp/month)	Annualized Sharpe ratio	One-way turnover (% NAV/month)
Free153	Firm size	Unhedged	17.60	231.41	0.26	7.73
Free153	Momentum	Coefficient	42.99	251.31	0.59	23.81
Free153	Momentum	Stock exposure	21.91	93.30	0.81	35.71
Free153	Momentum	Unhedged	35.98	262.05	0.48	21.04
Free36	Book-to-market	Coefficient	18.26	185.19	0.34	12.99
Free36	Book-to-market	Stock exposure	12.48	77.25	0.56	17.59
Free36	Book-to-market	Unhedged	18.20	191.98	0.33	10.92
Free36	Firm size	Coefficient	12.90	178.30	0.25	25.80
Free36	Firm size	Stock exposure	5.76	88.24	0.23	28.28
Free36	Firm size	Unhedged	17.60	231.41	0.26	7.73
Free36	Momentum	Coefficient	25.24	240.59	0.36	27.16
Free36	Momentum	Stock exposure	6.58	89.68	0.25	34.82
Free36	Momentum	Unhedged	35.98	262.05	0.48	21.04
Zero153	Book-to-market	Coefficient	21.36	180.60	0.41	12.74
Zero153	Book-to-market	Stock exposure	3.70	81.07	0.16	18.19
Zero153	Book-to-market	Unhedged	18.20	191.98	0.33	10.92
Zero153	Firm size	Coefficient	5.44	200.21	0.09	10.84
Zero153	Firm size	Stock exposure	-6.62	82.19	-0.28	25.87
Zero153	Firm size	Unhedged	17.60	231.41	0.26	7.73
Zero153	Momentum	Coefficient	39.33	251.40	0.54	23.42
Zero153	Momentum	Stock exposure	0.56	94.80	0.02	34.93
Zero153	Momentum	Unhedged	35.98	262.05	0.48	21.04
Zero36	Book-to-market	Coefficient	8.12	180.71	0.16	14.40
Zero36	Book-to-market	Stock exposure	4.29	76.86	0.19	18.66
Zero36	Book-to-market	Unhedged	18.20	191.98	0.33	10.92
Zero36	Firm size	Coefficient	7.54	179.06	0.15	24.26
Zero36	Firm size	Stock exposure	11.16	87.98	0.44	27.79
Zero36	Firm size	Unhedged	17.60	231.41	0.26	7.73
Zero36	Momentum	Coefficient	20.15	253.97	0.27	25.95
Zero36	Momentum	Stock exposure	-11.26	93.04	-0.42	34.47
Zero36	Momentum	Unhedged	35.98	262.05	0.48	21.04

Matching the characteristic exposure in each formation month changes the gross-position requirement, rather than imposing the same capital constraint. The matched-exposure table therefore reports gross positions together with volatility. Three formation months in the zero-intercept 36-characteristic size comparison require a sign reversal to match the coefficient portfolio's characteristic exposure. These observations are retained. The resulting control compares equal signed exposures without selecting only the months in which both original signals agree.

Table IA.41: Stock positions required to match the coefficient portfolio's signed exposure

Source intercept / no. of characteristics	Target	Mean gross / NAV	95th percentile gross/NAV	Volatility (bp/month)	Variance reduction vs. coefficient (%)	Sign reversals (months)
Unrestricted, 153	Book-to-market	1.27	1.42	107.09	65.00	0
Unrestricted, 153	Firm size	1.86	2.14	147.65	48.36	0
Unrestricted, 153	Momentum	1.54	1.73	144.18	67.09	0
Unrestricted, 36	Book-to-market	1.38	1.53	107.29	66.43	0
Unrestricted, 36	Firm size	1.58	2.28	160.50	18.97	0
Unrestricted, 36	Momentum	1.94	2.25	164.96	52.99	0
Zero intercept, 153	Book-to-market	1.27	1.40	104.04	66.81	0
Zero intercept, 153	Firm size	1.81	2.10	146.84	46.21	0
Zero intercept, 153	Momentum	1.61	1.80	150.72	64.05	0
Zero intercept, 36	Book-to-market	1.37	1.52	107.20	64.81	0
Zero intercept, 36	Firm size	1.47	2.26	156.93	23.19	3
Zero intercept, 36	Momentum	2.02	2.52	173.32	53.43	0

The stock-exposure projection is scaled to the coefficient portfolio's signed characteristic exposure in each month. This comparison holds that exposure fixed while allowing gross holdings to differ. Mean and 95th-percentile gross positions, in multiples of NAV, use all 225 formation months; volatility in monthly basis points and variance reduction use the 216 common return months. Positive variance reduction means lower variance than the corresponding coefficient portfolio. Sign flips count negative formation-time scaling ratios. Source intercept and library identify the estimated model; the underlying size profile is larger-firm positive. These are descriptive holdings and risk comparisons.

To calculate turnover, let $w_{i,t-1}$ be the preceding stock weight and $R_{i,t}$ its total return. With a cash position financing the zero-net-stock portfolio, portfolio wealth grows by $1 + r_{f,t} + \sum_i w_{i,t-1}(R_{i,t} - r_{f,t})$. The pretrade stock weight is the preceding weight multiplied by $1 + R_{i,t}$ and divided by that wealth factor. One-way turnover is one-half the sum of absolute changes from these pretrade weights to the new holdings, including entries and exits. We do not calculate turnover when a preceding nonzero holding lacks a return. The resulting 219 turnover months differ from the 216 common months used for the primary return comparison.

The replication files also report constant trading-cost scenarios of zero, five, ten, and twenty basis points per dollar traded. Each subtracts twice one-way turnover times the assumed cost from

the gross portfolio excess return, on common months with observed turnover. These calculations translate a trading requirement into a common cost scale. They do not estimate security-specific spreads, borrow fees, financing terms, or market impact.

Table IA.42: Mean excess returns under illustrative trading-cost assumptions

Target	Holdings rule	Months	0 bp cost	5 bp cost	10 bp cost	20 bp cost
Book-to-market	Coefficient	210	27.31	26.00	24.70	22.08
Book-to-market	Stock exposure	210	12.72	10.88	9.04	5.35
Firm size	Coefficient	210	8.99	8.01	7.02	5.05
Firm size	Stock exposure	210	-8.95	-11.60	-14.24	-19.54
Momentum	Coefficient	210	42.79	40.41	38.03	33.28
Momentum	Stock exposure	210	22.31	18.73	15.16	8.00

The columns show how a common assumed cost per dollar traded changes portfolio mean excess returns for the unrestricted 153-characteristic model at unit gross. Entries are monthly basis points after the assumed one-way cost, applied to both sides of one-way turnover. The columns hold scoring months fixed. Larger assumed costs reduce the displayed payoff according to the observed trading requirement. Costs are scenarios, not estimates of security-specific execution costs, financing, market impact, or borrowing charges.

Finally, we regress each realized stock-portfolio excess return on FFC6 over the evaluation sample to describe remaining return-factor exposure. This is an ex-post diagnostic, not an input to the holdings. For the unrestricted 153-characteristic model, centered FFC6 regression R^2 falls from 72.51% to 15.72% for value and from 67.75% to 17.84% for momentum under stock-exposure projection. The diagnostic supports a reduction in familiar factor exposure while showing why estimated loading neutrality and realized return-covariance neutrality should be evaluated separately. Full coefficients and calendar-preserving HAC standard errors based on Newey and West (1987) are included in the replication results.

Table IA.43: Realized factor exposure of the stock portfolios

Target	Holdings rule	Centered R^2 (%)	Market beta	Momentum beta	Alpha (bp/month)	HAC standard error (bp/month)
Book-to-market	Coefficient	72.51	-0.02	-0.24	27.06	6.64
Momentum	Coefficient	67.75	-0.07	0.43	43.21	9.47
Firm size	Coefficient	23.75	0.04	0.15	-2.60	14.29
Book-to-market	Stock exposure	15.72	-0.01	-0.04	14.69	6.23
Momentum	Stock exposure	17.84	0.05	0.09	17.44	7.44
Firm size	Stock exposure	11.86	0.05	0.03	-12.83	4.53

The table regresses subsequent portfolio excess returns on FFC6 to describe remaining return-factor exposure under the unrestricted 153-characteristic model. Centered regression R-squared is in percent; a smaller value means less realized variation explained by FFC6 and differs from the paper's uncentered Total R-squared. Alpha and its standard error are monthly basis points. Regressions use 216 common return months, and HAC uses 12 calendar lags with gaps retained. The regressions are ex-post diagnostics and do not determine holdings.

IA.10 Market Conditions and Risk Monitoring

The market-state variable is the standard deviation of the preceding twelve market returns, ending before formation. A high-volatility month exceeds the 75th percentile of up to 120 earlier volatility observations, with at least 24 observations required. This definition uses past information and is held fixed across models. Table IA.44 reports realized hedge variance reduction in the two groups. The conditional samples differ in both risk levels and dates, so the state comparison describes performance rather than identifying a causal effect of market volatility.

For the risk-monitoring experiment, let $L = [C, B]$, let $G = L'QL$ use the annual training-window reference matrix, and define $d_t = \|G^{-1/2}L'Q_tLG^{-1/2} - I_9\|_F / \sqrt{9}$. Here G is a Gram matrix of the selected loading coefficients, and the subscript F denotes the Frobenius norm, the square root of the sum of squared matrix entries. The matrix I_9 is the identity in those loading coordinates. This statistic records changes in the characteristic second moment along the nine selected loading directions. It does not measure changes in the full characteristic matrix outside that space. The forecast adds $\log(1 + d_t)$ to an intercept, log trailing market volatility, and log past hedge loss. Regressors are standardized using the estimation history available at each forecast date.

The outcome is the logarithm of mean squared target-hedge residuals over the next one, three, or twelve months. Training begins only after at least 60 outcomes have fully matured, and the forecast history expands thereafter. A horizon requiring a missing return is not scored. This yields 144, 122, and 41 forecast months at the respective horizons. Table IA.46 reports the reduction in

Table IA.44: Hedge variance reduction in high- and lower-volatility markets

Source intercept / no. of characteristics	Volatility state	Return months	Hedge variance reduction (%)
Free36	high	52	75.64
Free36	lower	164	65.75
Free153	high	52	73.31
Free153	lower	164	62.04
Zero36	high	52	75.13
Zero36	lower	164	65.12
Zero153	high	52	73.09
Zero153	lower	164	62.36

Each row uses FFC6 plus three latent hedge instruments. Entries are percentage reductions in the sum of individual target-return variances relative to the unhedged targets; larger values mean a greater fraction removed within the stated state, not necessarily a lower residual variance level across states. Market states use only trailing market returns at formation. The common return sample contains 52 high-volatility and 164 lower-volatility months. Free and Zero refer to source characteristic intercepts. These state comparisons are descriptive, with different market-risk levels and dates.

Table IA.45: Overlap among stock-characteristic profiles across market states

Characteristics	Volatility state	Formation months	Effective number of directions	Top-five eigenvalue share (%)
36	High	54	9.64	59.99
36	Lower	171	10.40	58.30
153	High	54	19.85	39.89
153	Lower	171	21.48	38.48

The spectra describe the cross-sectional overlap of standardized stock-characteristic profiles in each market state. They use uncentered characteristic second moments, excluding the constant. A larger top-five share indicates more concentration in a few characteristic directions; participation dimension summarizes how broadly the spectrum is distributed. Top-five share is in percent. Formation counts differ from return-score counts because no future-return information enters the characteristic matrix. These are descriptive exposure quantities, not shares of subsequent return variance.

squared log-loss forecast error from adding exposure drift. The outcome and its benchmark are distinct from return predictive R^2 and from a percentage reduction in portfolio variance.

Table IA.46: Does exposure drift improve forecasts of future hedge loss?

Source intercept / no. of characteristics	Horizon (months)	Forecast months	Log-loss forecast MSE reduction (%)	Lower 95% endpoint	Upper 95% endpoint
Unrestricted, 36	1	144	-4.44	-20.03	9.29
Unrestricted, 36	3	122	-11.15	-52.43	16.42
Unrestricted, 36	12	41	-57.45	-329.69	70.33
Unrestricted, 153	1	144	4.89	-7.03	13.91
Unrestricted, 153	3	122	11.29	-9.19	23.17
Unrestricted, 153	12	41	-12.16	-199.61	39.08
Zero intercept, 36	1	144	-5.86	-22.20	8.20
Zero intercept, 36	3	122	-18.28	-62.23	13.67
Zero intercept, 36	12	41	-21.80	-203.32	69.23
Zero intercept, 153	1	144	4.06	-8.04	13.68
Zero intercept, 153	3	122	10.57	-11.12	23.70
Zero intercept, 153	12	41	0.57	-102.97	42.05

The comparison adds loading-space exposure drift to forecasts based on past market volatility and past hedge loss. Estimates and interval endpoints are percentages. Positive values indicate a reduction in the mean squared error of log-loss forecasts relative to that benchmark. All paired intervals for the incremental effect include zero. The outcome is future hedge loss, not a portfolio return or a variance-reduction percentage. Forecast horizons use their own matured-outcome samples; the stored results report the conditional paired intervals for each horizon and model.

IA.11 Additional Partial-Hedging Results

Table IA.47 reports the half and full loading hedges for all four source-model specifications and all three economic targets. Exposure is the mean fraction of the unhedged portfolio's signed characteristic exposure retained at unit gross. Gross and matched volatility correspond to restoring that unhedged exposure each month. The remaining quarter and three-quarter rules, monthly holdings-based returns, trading requirements, and position concentration are included in the replication results.

Table IA.48 reports paired intervals for full hedging at the same desired characteristic exposure in the unrestricted 153-characteristic model. The changes are relative to the original unhedged portfolio, so negative numbers indicate lower subsequent volatility. The corresponding gross-

Table IA.47: Partial loading hedges: retained exposure and required stock positions

Source intercept / no. of characteristics	Target	Projection removed (%)	Unit-gross exposure retained (%)	Unit-gross volatility (bp/month)	Matched gross / NAV	Matched volatility (bp/month)
Unrestricted, 36	Book-to-market	50.00	94.02	142.87	1.06	152.85
Unrestricted, 36	Book-to-market	100.00	68.83	77.25	1.46	115.29
Unrestricted, 36	Momentum	50.00	94.52	227.31	1.06	238.57
Unrestricted, 36	Momentum	100.00	43.03	89.68	2.34	204.26
Unrestricted, 36	Firm size	50.00	97.19	205.47	1.03	212.21
Unrestricted, 36	Firm size	100.00	31.17	88.24	3.27	275.30
Unrestricted, 153	Book-to-market	50.00	94.62	143.32	1.06	152.47
Unrestricted, 153	Book-to-market	100.00	75.80	83.09	1.32	112.47
Unrestricted, 153	Momentum	50.00	93.54	205.60	1.07	218.64
Unrestricted, 153	Momentum	100.00	62.05	93.30	1.62	151.25
Unrestricted, 153	Firm size	50.00	95.61	186.43	1.05	195.34
Unrestricted, 153	Firm size	100.00	52.34	79.99	1.92	152.12
Zero intercept, 36	Book-to-market	50.00	93.93	144.01	1.06	154.13
Zero intercept, 36	Book-to-market	100.00	68.35	76.86	1.47	115.09
Zero intercept, 36	Momentum	50.00	94.59	227.45	1.06	238.51
Zero intercept, 36	Momentum	100.00	41.75	93.04	2.42	213.32
Zero intercept, 36	Firm size	50.00	97.18	207.00	1.03	213.83
Zero intercept, 36	Firm size	100.00	31.30	87.98	3.27	273.32
Zero intercept, 153	Book-to-market	50.00	94.59	143.23	1.06	152.24
Zero intercept, 153	Book-to-market	100.00	75.70	81.07	1.32	109.24
Zero intercept, 153	Momentum	50.00	93.48	207.57	1.07	220.59
Zero intercept, 153	Momentum	100.00	59.10	94.80	1.70	158.28
Zero intercept, 153	Firm size	50.00	95.58	184.92	1.05	193.80
Zero intercept, 153	Firm size	100.00	53.32	82.19	1.89	152.74

Each row removes the stated fraction of the current loading projection while holding gross at one, then separately scales the residual portfolio back to the original characteristic exposure. Hedge and retained exposure are percentages; volatility is in monthly basis points. A larger hedge fraction is an imposed loading restriction, not a percentage reduction in risk. Gross/NAV is the average absolute stock position after matching the original unhedged characteristic exposure; matched volatility refers to that scaled portfolio. Formation quantities use 225 months and return quantities use 216 common months. Source-intercept restrictions and characteristic counts identify the fitted models; size is larger-firm positive. Entries are descriptive outcomes of the fixed hedge rules.

position requirements appear in the main-text table. The replication files retain the same contrasts for both libraries, both intercept restrictions, and half hedging.

Table IA.48: Full loading hedges: volatility changes at the original exposure

Target	Volatility change (bp/month)	Lower 95% endpoint	Upper 95% endpoint
Book-to-market	-79.52	-129.40	-36.63
Firm size	-79.29	-116.35	-34.56
Momentum	-110.80	-174.68	-45.02

Entries are the full-hedge portfolio's monthly volatility minus the original unhedged portfolio's volatility, in basis points, under the unrestricted 153-characteristic model. Negative values favor the full hedge. Both rules have the same signed characteristic exposure at each formation date; gross holdings can differ. Pointwise 95% intervals use 1,999 paired twelve-month calendar-block draws conditional on the fitted models and evaluated payoff paths. These contrasts are separate from the three simultaneous risk families.

IA.12 Complete Exposure-Update and Covariance Results

The complete tables retain every economic anchor, source-model specification, and portfolio rule. Mean excess returns and volatility are in monthly basis points on 216 common returns. Gross holdings and retained characteristic exposure average 225 formation months; turnover is one-way monthly percent on 219 observations. Retention expresses signed characteristic exposure as a percentage of the current unhedged unit-gross portfolio's exposure. The current and covariance rules labeled "positive anchor" both have exactly 100% retention in every month; their gross holdings differ.

The covariance-correction comparison fixes the initial characteristic coefficient and changes its loading-space hedge using historical return covariance. It is distinct from the positive-anchor covariance rule, which chooses all coefficients in the span of the characteristic and loading profiles subject to explicit cash and characteristic constraints. The latter is the direct minimum-variance comparison for a common positive exposure. The correction and matched-signed-exposure comparisons remain in the table to document how the economic restriction affects the result.

Some frozen or covariance-correction portfolios have negative signed characteristic exposure. Their matched current-rule controls preserve that sign: there are 29 such formation observations across all frozen comparisons and 21 across all covariance-correction comparisons. They are retained in both point estimates and uncertainty calculations. The fixed-positive-exposure comparison has no sign reversals. In the uncertainty table, positive variance reduction favors the current-

rule portfolio; negative values favor its stated comparator. Intervals are pointwise and conditional on the estimated models, with no claim of joint significance across all rows.

Table IA.49: All exposure-update and covariance portfolios

Mean excess returns and volatility use 216 common subsequent months. Gross/NAV and signed tilt retained average 225 formation months; one-way turnover uses 219 eligible months. Tilt retained is percent of the original unhedged unit-gross characteristic exposure. Rules without an anchor have unit gross; anchored rules scale current-loading-neutral holdings to the named rule's signed tilt or to the original positive tilt. Cov. positive anchor instead minimizes historical variance at that positive target. Sign flips count negative characteristic exposures. Free/Zero identify source intercepts and 36/153 count characteristics; Value is book-to-market and Size is larger-firm positive.

Source intercept / no. of char.	Target	Rule	Mean excess return (bp/month)	Volatility (bp/month)	Gross / NAV	Turnover (% NAV/ month)	Tilt retained (%)	Sign flips (months)
Free36	Value	Coefficient	18.26	185.19	1.00	12.99	94.70	0
Free36	Value	Current	12.48	77.25	1.00	17.59	68.83	0
Free36	Value	Training	11.13	97.47	1.00	16.92	70.92	0
Free36	Value	Cov. correction	14.20	86.74	1.00	15.93	73.73	0
Free36	Value	Current / training an- chor	12.87	79.71	1.03	18.16	70.92	0
Free36	Value	Current / cov. anchor	12.70	82.23	1.07	18.84	73.73	0
Free36	Value	Current / positive an- chor	18.32	115.29	1.46	25.89	100.00	0
Free36	Value	Cov. / positive anchor	24.70	131.53	1.23	18.86	100.00	0
Free36	Momentum	Coefficient	25.24	240.59	1.00	27.16	83.04	0
Free36	Momentum	Current	6.58	89.68	1.00	34.82	43.03	0
Free36	Momentum	Training	9.32	120.00	1.00	33.82	46.52	3
Free36	Momentum	Cov. correction	8.15	117.90	1.00	33.60	47.29	0
Free36	Momentum	Current / training an- chor	5.79	95.26	1.07	38.26	46.52	3
Free36	Momentum	Current / cov. anchor	6.63	95.11	1.09	38.66	47.29	0
Free36	Momentum	Current / positive an- chor	18.30	204.26	2.34	81.40	100.00	0
Free36	Momentum	Cov. / positive anchor	15.63	240.14	1.42	45.31	100.00	0
Free36	Size	Coefficient	12.90	178.30	1.00	25.80	50.92	0
Free36	Size	Current	5.76	88.24	1.00	28.28	31.17	0
Free36	Size	Training	-7.73	133.59	1.00	27.72	20.49	10

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Table IA.49: All exposure-update and covariance portfolios

Source intercept / no. of char.	Target	Rule	Mean excess return (bp/month)	Volatility (bp/month)	Gross / NAV	Turnover (% NAV/ month)	Tilt retained (%)	Sign flips (months)
Free36	Size	Cov. correction	0.61	116.62	1.00	26.06	29.07	10
Free36	Size	Current / training an- chor	6.23	61.96	0.68	20.79	20.49	10
Free36	Size	Current / cov. anchor	8.82	89.08	0.95	27.99	29.07	10
Free36	Size	Current / positive an- chor	12.85	275.30	3.27	92.99	100.00	0
Free36	Size	Cov. / positive anchor	-7.65	185.25	1.35	23.57	100.00	0
Free153	Value	Coefficient	24.49	181.03	1.00	13.05	96.22	0
Free153	Value	Current	12.93	83.09	1.00	18.40	75.80	0
Free153	Value	Training	13.33	117.13	1.00	17.96	76.04	0
Free153	Value	Cov. correction	16.90	102.85	1.00	17.77	76.50	0
Free153	Value	Current / training an- chor	13.46	82.86	1.00	18.47	76.04	0
Free153	Value	Current / cov. anchor	12.92	83.54	1.01	18.64	76.50	0
Free153	Value	Current / positive an- chor	17.03	112.47	1.32	24.56	100.00	0
Free153	Value	Cov. / positive anchor	25.52	141.18	1.21	20.03	100.00	0
Free153	Momentum	Coefficient	42.99	251.31	1.00	23.81	94.95	0
Free153	Momentum	Current	21.91	93.30	1.00	35.71	62.05	0
Free153	Momentum	Training	21.40	123.35	1.00	34.90	66.65	0
Free153	Momentum	Cov. correction	22.25	118.47	1.00	34.60	60.12	0
Free153	Momentum	Current / training an- chor	23.42	96.74	1.08	38.31	66.65	0
Free153	Momentum	Current / cov. anchor	21.26	85.26	0.97	34.47	60.12	0
Free153	Momentum	Current / positive an- chor	34.42	151.25	1.62	57.84	100.00	0
Free153	Momentum	Cov. / positive anchor	34.47	228.99	1.33	42.44	100.00	0
Free153	Size	Coefficient	9.23	205.45	1.00	9.87	96.91	0
Free153	Size	Current	-8.54	79.99	1.00	26.52	52.34	0
Free153	Size	Training	-17.73	108.86	1.00	26.83	43.79	0
Free153	Size	Cov. correction	-15.22	104.73	1.00	27.01	45.22	0

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Table IA.49: All exposure-update and covariance portfolios

Source intercept / no. of char.	Target	Rule	Mean excess return (bp/month)	Volatility (bp/month)	Gross / NAV	Turnover (% NAV/ month)	Tilt retained (%)	Sign flips (months)
Free153	Size	Current / training an- chor	-7.68	68.15	0.83	22.25	43.79	0
Free153	Size	Current / cov. anchor	-8.15	72.46	0.85	22.88	45.22	0
Free153	Size	Current / positive an- chor	-15.66	152.12	1.92	51.29	100.00	0
Free153	Size	Cov. / positive anchor	-10.94	171.25	1.28	24.86	100.00	0
Zero36	Value	Coefficient	8.12	180.71	1.00	14.40	93.48	0
Zero36	Value	Current	4.29	76.86	1.00	18.66	68.35	0
Zero36	Value	Training	2.96	96.07	1.00	18.17	69.54	0
Zero36	Value	Cov. correction	10.11	88.60	1.00	16.50	72.67	0
Zero36	Value	Current / training an- chor	4.21	77.65	1.02	18.99	69.54	0
Zero36	Value	Current / cov. anchor	4.39	80.86	1.06	19.75	72.67	0
Zero36	Value	Current / positive an- chor	6.23	115.09	1.47	27.62	100.00	0
Zero36	Value	Cov. / positive anchor	25.35	133.94	1.24	19.14	100.00	0
Zero36	Momentum	Coefficient	20.15	253.97	1.00	25.95	83.57	0
Zero36	Momentum	Current	-11.26	93.04	1.00	34.47	41.75	0
Zero36	Momentum	Training	-15.10	128.17	1.00	33.17	44.74	1
Zero36	Momentum	Cov. correction	-14.89	128.70	1.00	33.66	42.95	2
Zero36	Momentum	Current / training an- chor	-11.37	98.15	1.07	37.51	44.74	1
Zero36	Momentum	Current / cov. anchor	-10.66	95.02	1.02	36.23	42.95	2
Zero36	Momentum	Current / positive an- chor	-24.74	213.32	2.42	83.17	100.00	0
Zero36	Momentum	Cov. / positive anchor	3.94	252.19	1.42	44.15	100.00	0
Zero36	Size	Coefficient	7.54	179.06	1.00	24.26	48.17	3
Zero36	Size	Current	11.16	87.98	1.00	27.79	31.30	0
Zero36	Size	Training	-0.37	139.00	1.00	27.45	20.86	15
Zero36	Size	Cov. correction	6.61	124.90	1.00	24.96	29.76	9
Zero36	Size	Current / training an- chor	7.48	62.09	0.70	20.92	20.86	15

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Table IA.49: All exposure-update and covariance portfolios

Source intercept / no. of char.	Target	Rule	Mean excess return (bp/month)	Volatility (bp/month)	Gross / NAV	Turnover (% NAV/ month)	Tilt retained (%)	Sign flips (months)
Zero36	Size	Current / cov. anchor	10.79	89.72	0.95	27.35	29.76	9
Zero36	Size	Current / positive anchor	31.60	273.32	3.27	91.69	100.00	0
Zero36	Size	Cov. / positive anchor	-11.92	179.75	1.34	22.57	100.00	0
Zero153	Value	Coefficient	21.36	180.60	1.00	12.74	96.27	0
Zero153	Value	Current	3.70	81.07	1.00	18.19	75.70	0
Zero153	Value	Training	7.01	115.46	1.00	17.97	75.69	0
Zero153	Value	Cov. correction	15.63	104.82	1.00	17.19	77.50	0
Zero153	Value	Current / training anchor	4.05	80.52	1.00	18.22	75.69	0
Zero153	Value	Current / cov. anchor	3.60	82.40	1.02	18.68	77.50	0
Zero153	Value	Current / positive anchor	4.79	109.24	1.32	24.26	100.00	0
Zero153	Value	Cov. / positive anchor	25.48	142.76	1.21	19.73	100.00	0
Zero153	Momentum	Coefficient	39.33	251.40	1.00	23.42	94.60	0
Zero153	Momentum	Current	0.56	94.80	1.00	34.93	59.10	0
Zero153	Momentum	Training	-1.54	120.78	1.00	34.28	63.10	0
Zero153	Momentum	Cov. correction	-0.24	118.53	1.00	35.12	56.06	0
Zero153	Momentum	Current / training anchor	0.01	97.89	1.07	37.24	63.10	0
Zero153	Momentum	Current / cov. anchor	0.59	86.06	0.95	33.08	56.06	0
Zero153	Momentum	Current / positive anchor	-1.60	158.28	1.70	59.38	100.00	0
Zero153	Momentum	Cov. / positive anchor	22.25	228.61	1.33	42.97	100.00	0
Zero153	Size	Coefficient	5.44	200.21	1.00	10.84	95.85	0
Zero153	Size	Current	-6.62	82.19	1.00	25.87	53.32	0
Zero153	Size	Training	-19.99	113.23	1.00	26.39	41.66	0
Zero153	Size	Cov. correction	-18.64	103.98	1.00	26.46	44.60	0
Zero153	Size	Current / training anchor	-5.44	65.18	0.78	20.48	41.66	0
Zero153	Size	Current / cov. anchor	-6.58	72.37	0.82	21.60	44.60	0

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Table IA.49: All exposure-update and covariance portfolios

Source intercept / no. of char.	Target	Rule	Mean excess return (bp/month)	Volatility (bp/month)	Gross / NAV	Turnover (% NAV/ month)	Tilt retained (%)	Sign flips (months)
Zero153	Size	Current / positive anchor	-11.00	152.74	1.89	49.30	100.00	0
Zero153	Size	Cov. / positive anchor	-11.01	172.05	1.27	24.55	100.00	0

Table IA.50: Paired uncertainty for all portfolio-risk comparisons

Entries are percentage reductions in subsequent return variance under the Current rule relative to the named benchmark, over 216 common return months. Positive values favor Current. Gross compares unit-gross portfolios; matched scales Current to the benchmark's signed characteristic exposure, including negatives; positive anchor matches the original unhedged positive exposure in both rules. Lower and upper columns are pointwise 95% paired intervals from 1,999 twelve-month calendar-block draws, conditional on fitted models and observed paths. They are separate from the simultaneous risk families. Free/Zero identify source intercepts and 36/153 count characteristics; Value is book-to-market and Size is larger-firm positive.

Source intercept / no. of characteristics	Characteristic	Benchmark	Variance reduction (%)	Lower 95% endpoint	Upper 95% endpoint
Free36	Value	Training, gross	37.18	19.60	52.05
Free36	Value	Training, matched	33.12	12.56	51.54
Free36	Value	Cov. correction, gross	20.67	8.32	29.96
Free36	Value	Cov. correction, matched	10.12	-5.39	22.65
Free36	Value	Cov. positive anchor	23.17	-11.48	48.78
Free36	Momentum	Training, gross	44.15	25.00	62.61
Free36	Momentum	Training, matched	36.98	12.68	62.03
Free36	Momentum	Cov. correction, gross	42.14	9.60	65.67
Free36	Momentum	Cov. correction, matched	34.92	1.38	61.75
Free36	Momentum	Cov. positive anchor	27.65	-38.42	60.99
Free36	Size	Training, gross	56.38	44.91	64.48
Free36	Size	Training, matched	78.49	68.59	86.71
Free36	Size	Cov. correction, gross	42.76	25.97	56.38
Free36	Size	Cov. correction, matched	41.66	10.14	65.87
Free36	Size	Cov. positive anchor	-120.86	-205.96	-72.86
Free153	Value	Training, gross	49.68	19.69	65.30

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Table IA.50: Paired uncertainty for all portfolio-risk comparisons

Source intercept / no. of characteristics	Characteristic	Benchmark	Variance reduction (%)	Lower 95% endpoint	Upper 95% endpoint
Free153	Value	Training, matched	49.95	18.33	66.00
Free153	Value	Cov. correction, gross	34.73	10.35	51.86
Free153	Value	Cov. correction, matched	34.02	9.86	49.68
Free153	Value	Cov. positive anchor	36.54	-4.11	60.86
Free153	Momentum	Training, gross	42.79	25.68	58.17
Free153	Momentum	Training, matched	38.50	22.02	52.53
Free153	Momentum	Cov. correction, gross	37.98	23.47	50.31
Free153	Momentum	Cov. correction, matched	48.20	38.08	57.96
Free153	Momentum	Cov. positive anchor	56.37	32.39	70.05
Free153	Size	Training, gross	46.00	28.77	60.03
Free153	Size	Training, matched	60.81	45.31	73.59
Free153	Size	Cov. correction, gross	41.66	28.39	53.74
Free153	Size	Cov. correction, matched	52.13	37.10	66.37
Free153	Size	Cov. positive anchor	21.10	-14.01	38.48
Zero36	Value	Training, gross	35.99	20.09	50.06
Zero36	Value	Training, matched	34.66	15.64	50.75
Zero36	Value	Cov. correction, gross	24.75	12.61	34.52
Zero36	Value	Cov. correction, matched	16.72	-0.21	29.66
Zero36	Value	Cov. positive anchor	26.16	-5.91	47.24
Zero36	Momentum	Training, gross	47.31	29.39	61.48
Zero36	Momentum	Training, matched	41.35	21.81	58.78
Zero36	Momentum	Cov. correction, gross	47.74	17.20	65.66
Zero36	Momentum	Cov. correction, matched	45.49	16.87	64.05
Zero36	Momentum	Cov. positive anchor	28.45	-26.04	56.91
Zero36	Size	Training, gross	59.94	48.52	67.53
Zero36	Size	Training, matched	80.05	70.39	87.20
Zero36	Size	Cov. correction, gross	50.38	35.50	61.79
Zero36	Size	Cov. correction, matched	48.39	18.42	70.22
Zero36	Size	Cov. positive anchor	-131.20	-221.39	-81.21
Zero153	Value	Training, gross	50.70	24.10	66.30
Zero153	Value	Training, matched	51.37	25.12	66.36
Zero153	Value	Cov. correction, gross	40.18	16.81	55.91

Continued on next page

Table IA.50: Paired uncertainty for all portfolio-risk comparisons

Source intercept / no. of characteristics	Characteristic	Benchmark	Variance reduction (%)	Lower 95% endpoint	Upper 95% endpoint
Zero153	Value	Cov. correction, matched	38.20	14.73	53.49
Zero153	Value	Cov. positive anchor	41.45	1.85	63.74
Zero153	Momentum	Training, gross	38.39	20.28	55.93
Zero153	Momentum	Training, matched	34.31	15.98	51.82
Zero153	Momentum	Cov. correction, gross	36.03	19.71	50.02
Zero153	Momentum	Cov. correction, matched	47.29	33.76	60.83
Zero153	Momentum	Cov. positive anchor	52.06	21.38	68.05
Zero153	Size	Training, gross	47.31	30.27	59.89
Zero153	Size	Training, matched	66.87	54.51	76.42
Zero153	Size	Cov. correction, gross	37.52	24.68	49.53
Zero153	Size	Cov. correction, matched	51.56	36.81	65.02
Zero153	Size	Cov. positive anchor	21.18	-17.42	39.88

IA.13 Characteristic Definitions

The dictionary below gives the source-library economic name of every characteristic and identifies the fixed 36-characteristic subset. The main-text group labels organize economic measurements for reading; they are neither estimated factor labels nor evidence that each group has a distinct risk premium. The machine-readable dictionary preserves source names, characteristic codes, and group assignments. All transformations used in estimation are implemented in the replication code.

Table IA.51: Characteristic dictionary

Code	Economic measurement	Baseline 36
age	Firm age	
aliq_at	Liquidity of book assets	
aliq_mat	Liquidity of market assets	
ami_126d	Absolute return-to-volume	Yes
at_be	Book leverage	
at_gr1	Investment-to-assets	Yes
at_me	Assets-to-market	Yes
at_turnover	Capital turnover	
be_gr1a	Change in common equity	
be_me	Book-to-market equity	Yes

Code	Economic measurement	Baseline 36
beta_60m	The market beta	Yes
beta_dimson_21d	The Dimson beta	
betabab_1260d	The Frazzini-Pedersen beta	Yes
betadown_252d	Downside beta	
bev_mev	Enterprise book-to-price	
bidaskhl_21d	The high-low bid-ask spread	
capex_abn	Abnormal corporate investment	
capx_gr1	Investment growth	Yes
capx_gr2	2-year investment growth	
capx_gr3	3-year investment growth	
cash_at	Cash-to-assets	
chcsho_12m	Net stock issues	
coa_gr1a	Change in current operating assets	
col_gr1a	Change in current operating liabilities	
cop_at	Cash-based operating profits-to-assets	Yes
cop_atl1	Cash-based operating profits-to-lagged book assets	
corr_1260d	Market Correlation	
coskew_21d	Coskewness	
cowc_gr1a	Change in net noncash working capital	
dbnetis_at	Net debt finance	
debt_gr3	Composite debt issuance	
debt_me	Debt-to-market	Yes
dgp_dsale	Percentage change in gross margin minus percentage change in sales	
div12m_me	Dividend yield	
dolvol_126d	Dollar trading volume	Yes
dolvol_var_126d	Coefficient of variation for dollar trading volume	
dsale_dinv	Percentage change in sales minus percentage change in inventory	
dsale_drec	Percentage change in sales minus percentage change in accounts receivable	
dsale_dsga	Percentage change in sales minus percentage change in SG&A	
earnings_variability	Earnings Variability	
ebit_bev	EBIT-to-book enterprise value	Yes
ebit_sale	Profit margin	
ebitda_mev	EBITDA-to-enterprise market value	Yes
emp_gr1	Hiring rate	
eq_dur	Equity duration	
eqnetis_at	Net equity finance	

Code	Economic measurement	Baseline 36
eqnpo_12m	Composite equity issuance	
eqnpo_me	Net payout yield	
eqpo_me	Payout yield	
f_score	Fundamental score	Yes
fcf_me	Cash flow-to-price	
fnl_gr1a	Change in financial liabilities	
gp_at	Gross profits-to-assets	Yes
gp_atl1	Gross profits-to-lagged assets	
inv_gr1	Inventory growth	
inv_gr1a	Inventory changes	Yes
iskew_capm_21d	Idiosyncratic skewness from the CAPM	
iskew_ff3_21d	Idiosyncratic skewness from the Fama-French 3-factor model	
iskew_hxz4_21d	Idiosyncratic skewness from the q-factor model	
ival_me	Intrinsic value-to-market	
ivol_capm_21d	Idiosyncratic volatility from the CAPM	
ivol_capm_252d	Idiosyncratic volatility	Yes
ivol_ff3_21d	Idiosyncratic volatility from the Fama-French 3-factor model	
ivol_hxz4_21d	Idiosyncratic volatility from the q-factor model	
kz_index	Kaplan-Zingales index	
lnoa_gr1a	Change in long-term net operating assets	
lti_gr1a	Change in long-term investments	
market_equity	Size (the market equity)	Yes
mispricing_mgmt	Mispricing factor: Management	
mispricing_perf	Mispricing factor: Performance	
ncoa_gr1a	Change in noncurrent operating assets	
ncol_gr1a	Change in noncurrent operating liabilities	
netdebt_me	Net debt-to-price	Yes
netis_at	Net external finance	
nfna_gr1a	Change in net financial assets	
ni_ar1	Earnings Persistence	
ni_be	Annual Return on equity	Yes
ni_inc8q	Number of consecutive quarters with earnings increases	
ni_ivol	Net Income Idiosyncratic Volatility	
ni_me	Earnings-to-price	
niq_at	Return on assets	
niq_at_chg1	4-quarter change in return on assets	
niq_be	Return on equity	Yes
niq_be_chg1	4-quarter change in return on equity	

Code	Economic measurement	Baseline 36
niq_su	Earnings surprise	
nncoa_gr1a	Change in net noncurrent operating assets	
noa_at	Net operating assets	
noa_gr1a	Change in net operating assets	Yes
o_score	Ohlson's (1980) O-score	Yes
oaccruals_at	Operating accruals	Yes
oaccruals_ni	Percent operating accruals	
ocf_at	Operating cash flow to assets	
ocf_at_chg1	1 year change in operating cash flow to assets	
ocf_me	Operating cash flow-to-price	
ocfq_saleq_std	Cash flow volatility	
op_at	Operating profits-to-assets	
op_atl1	Operating profits-to-lagged book assets	
ope_be	Operating profits-to-book equity	Yes
ope_bel1	Operating profits-to-lagged book equity	
opex_at	Operating leverage proxy	
pi_nix	Taxable income-to-book income	
ppeinv_gr1a	Change in property, plant and equipment and equipment and inventory scaled by book assets	
prc	Price per share	
prc_highprc_252d	52-week high	
qmj	Quality minus Junk: Composite	Yes
qmj_growth	Quality minus Junk: Growth	
qmj_prof	Quality minus Junk: Profitability	
qmj_safety	Quality minus Junk: Safety	
rd_me	R&D expense-to-market	
rd_sale	R&D expense-to-sales	
rd5_at	R&D capital-to-book assets	
resff3_12_1	Residual momentum	Yes
resff3_6_1	Residual momentum	
ret_1_0	Short-term reversal	Yes
ret_12_1	Price momentum, prior 11-month returns	Yes
ret_12_7	Past 12 to 7 month momentum	Yes
ret_3_1	Past 3 month return	
ret_6_1	Price momentum, prior 6-month returns	Yes
ret_60_12	Long-term reversal	
ret_9_1	Past 9 month return	
rmax1_21d	Maximum daily return	
rmax5_21d	Highest 5 days of return	
rmax5_rvol_21d	Highest 5 days of return scaled by volatility	

Code	Economic measurement	Baseline 36
rskew_21d	Total skewness	
rvol_21d	Total volatility	Yes
sale_bev	Sales-to-book enterprise value	
sale_emp_gr1	Labor force efficiency	
sale_gr1	Annual sales growth	
sale_gr3	Sales Growth 3yr	
sale_me	Sales-to-price	Yes
saleq_gr1	Quarterly sales growth	
saleq_su	Revenue surprise	
seas_1_1an	Year 1-lagged return, annual	
seas_1_1na	Year 1-lagged return, nonannual	Yes
seas_11_15an	Years 11–15 lagged returns, annual	
seas_11_15na	Years 11–15 lagged returns, nonannual	
seas_16_20an	Years 16–20 lagged returns, annual	
seas_16_20na	Years 16–20 lagged returns, nonannual	
seas_2_5an	Years 2–5 lagged returns, annual	
seas_2_5na	Years 2–5 lagged returns, nonannual	
seas_6_10an	Years 6–10 lagged returns, annual	
seas_6_10na	Years 6–10 lagged returns, nonannual	
sti_gr1a	Change in short-term investments	
taccruals_at	Total accruals	Yes
taccruals_ni	Percent total accruals	
tangibility	Tangibility of assets	
tax_gr1a	Tax expense surprise	
turnover_126d	Share turnover	Yes
turnover_var_126d	Coefficient of variation for share turnover	
z_score	Altman's (1968) Z-score	Yes
zero_trades_126d	Prior 6-month turnover-adjusted number of zero daily trading volume	
zero_trades_21d	Prior 1-month turnover-adjusted number of zero daily trading volume	
zero_trades_252d	Prior 12-month turnover-adjusted number of zero daily trading volume	

IA.14 Reader's Notation Guide

The central investment calculation has three steps: specify the original characteristic exposure, measure the exposure left per unit gross after a loading hedge, and scale the remaining stock positions to recover the target.

Stocks and characteristics. Z_t has stocks in rows and characteristics, including the constant, in columns; n_t counts formation stocks. A direction a selects a characteristic, and $s_t = Z_t a$ is its profile across stocks.

Exposure ruler. $Q_t = Z_t' Z_t / n_t$ is the characteristic second moment. Training uses the estimation-window average, Origin the March snapshot, and Current the formation-month matrix. These matrices measure stock-profile overlap.

Model loadings. C and B map characteristics to observable and latent factor loadings. For the cash-inclusive portfolio projection, $L = [e_0, C, B]$ and $H_t = Z_t L$ include the constant and current stock-loading profiles. Coverage comparisons state their selected loading columns separately.

Original investment. $s_{c,t}$ is the demeaned characteristic profile. Its unit-gross portfolio w_t^0 has positive exposure ξ_t^0 , the target shared by the original-positive comparisons each month.

Remaining position. P_{H_t} projects onto the current loading profiles. In the gross identity, u_t is the residual characteristic profile and w_t^N its unit-gross portfolio. Partial hedging instead defines its residual from unit-gross initial holdings.

Exposure and gross. η_t is the positive exposure per unit gross for the nonzero Current projection. $G_t = \xi_t^0 / \eta_t$ gives its target-matched gross holdings divided by NAV. For a general reference M , η_t^M is signed exposure; matched gross uses its absolute value.

Holdings mechanism. $\mathcal{C}_t^{\text{cash}}$ is the fraction of the centered profile's squared norm removed by the cash-inclusive projection. ℓ_t compares residual and original absolute profile sizes. Their identity explains gross along the selected direction.

Returns and intercepts. g contains observable factor returns, f latent model factors, and p source-stock latent portfolio payoffs. Γ_α is the source-model characteristic intercept; $c_{j,t}$ is the separate target-regression intercept. $\hat{\lambda}_t$ contains historical factor means used in forecasts.

Performance. Total R^2 concerns realized-return fit; predictive R^2 concerns forecasts. RPE compares squared average pricing errors with squared average target payoffs. HVR aggregates variance reductions from factor-payoff hedges. Each portfolio variance comparison names its own reference.

Calendar and local notation. The inference appendix uses T_c for calendar positions and n for observed return months. Local G denotes a Gram matrix, not gross G_t . Unsubscripted η and θ have separately stated optimization or inference roles.

IA.15 Aggregation of Performance Measures

Total R^2 pools squared errors before taking their ratio. Two complementary summaries, following Kelly et al. (2023b), first calculate a ratio for each asset or each month. Write $e_{j,t+1}^v = y_{j,t+1} - q_{j,t+1}^v$ for the factor-component or complete-fit error. If asset j has T_j observed evaluation payoffs, average time-series R^2 is

$$R_{\text{TS},v}^2 = \frac{1}{\sum_j T_j} \sum_j T_j \left[1 - \frac{\sum_t (e_{j,t+1}^v)^2}{\sum_t y_{j,t+1}^2} \right]. \quad (\text{IA.15.1})$$

The common target panel is balanced, so this is the simple average of the target-specific statistics. Relative to total R^2 , it gives each target equal weight rather than weighting by its sum of squared payoffs.

Average cross-section R^2 is

$$R_{\text{CS},v}^2 = \frac{1}{T} \sum_t \left[1 - \frac{\sum_j (e_{j,t+1}^v)^2}{\sum_j y_{j,t+1}^2} \right]. \quad (\text{IA.15.2})$$

It gives each evaluation month equal weight. The slopes and factor payoffs are those used to evaluate the model, with no new cross-sectional slope fitted on evaluation returns. This statistic summarizes monthly realized-return fit; relative pricing error in (6) instead compares time averages across the test assets. All three realized-fit statistics use squared excess payoffs without demeaning.

When evaluating a source model directly on individual stocks, the complete fitted return includes its own estimated conditional intercept, $z'_{i,t} \hat{\Gamma}_\alpha$, along with the observable and latent factor terms. This is the unrestricted total-fit convention of Kelly et al. (2019). The fixed-target regressions in the main text have their own intercepts $c_{j,t}$ and slopes $b_{j,t}$. Separate labels preserve the

distinction between the original stock model, its factor compensation, and a complete regression fitted to new target payoffs.

The weighting of source-stock errors is also explicit. Let n_t be the number of source stocks entering the fit in month t , and let $e_{i,t+1}$ be the corresponding model residual. Equal weighting of stock observations gives the pooled statistic

$$R_{\text{source}}^2(p) = 1 - \frac{\sum_t p_t n_t^{-1} \sum_i e_{i,t+1}^2}{\sum_t p_t n_t^{-1} \sum_i r_{i,t+1}^2}, \quad p_t = \frac{n_t}{\sum_s n_s}. \quad (\text{IA.15.3})$$

An equal-month version instead sets $p_t = 1/T$. The two weightings coincide when the stock count is constant; otherwise the pooled version gives greater weight to months with more stocks. We retain the weighting used by each reported source fit and label equal-month comparisons separately. With a fixed number of target portfolios and common evaluation dates, the target-panel statistic in (5) has no such stock-count difference.