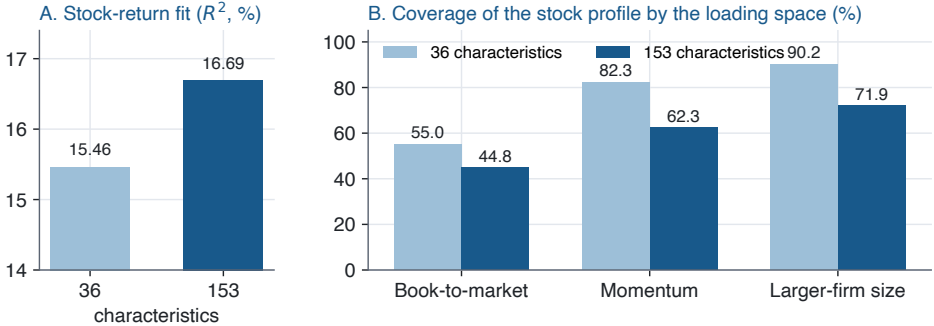
The Ruler Matters

Factor Exposures, Attribution, and Portfolio Risk

Mingyang Liu
Imperial College London

Job Market Talk • November 2026

More characteristics, better fit, less of the positions we care about



*Same model, same factors, same stocks. Adding 117 characteristics improves fit and **lowers** the share of value, momentum and size that the loading space represents.*

IPCA with FFC6 + 3 latent factors, re-estimated each March 2007–2025 on 240 months. Fit and coverage on the same training window. Direction holds at all 19 origins, for all three profiles, under both intercept specifications.

Why this matters: two people act on the same loadings

The researcher attributes

- How much of a fitted return is **observed factors**, **latent factors**, or a **characteristic intercept**?
- If those components load on the *same stocks*, the split is a choice.

The investor hedges

- Keep a momentum or value tilt, **neutralize** the estimated factor loadings.
- If the tilt and the loadings sit on the same stocks, neutralizing removes part of the tilt.

*Both decisions depend on **which stock positions the loadings represent**.*
Neither return fit nor coefficient values tell you that.

The question

Which stock positions does an estimated factor-loading space represent,
and what does that imply for attribution and for portfolios?

1. **See.** As the characteristic library grows, how much of value, momentum and size does a fixed-dimensional loading space cover?
2. **Credit.** When observed and latent loadings overlap on stocks, how much does the reported attribution move with fit held fixed?
3. **Hold.** To keep the same characteristic target, which holdings, gross positions and risks do different loading spaces require?
4. **Measure.** With coefficients held fixed, stocks keep changing. Which cross section should the restriction be measured on?

What I find

1. Fit $\neq$ coverage — Training R^2 **15.5% $\rightarrow$ 16.7%**, while momentum coverage falls **82% $\rightarrow$ 62%**. Every one of 19 origins.

2. Credit moves — Same fitted size forecast of **2.16 bp**: the observable share goes from **1.4** to **45.5 bp**, latent offsets it.

3. Holdings move — Same size target: gross **3.27 $\rightarrow$ 1.92** $\times$ NAV, volatility **275 $\rightarrow$ 152 bp** per month.

4. The ruler moves — Fixed coefficients, current cross section: **27–69% lower** variance than the annual-origin rule; 9 of 12 simultaneous intervals exclude zero.

The economic content of a factor model's loading space depends on how its stock exposures are measured.

Where this sits

Characteristic-based and conditional factor models

Kelly, Pruitt and Su (2019); Gu, Kelly and Xiu (2021); Lettau and Pelger (2020); Giglio and Xiu (2021); Kozak, Nagel and Santosh (2018, 2020); Freyberger, Neuhierl and Weber (2020); Jensen, Kelly and Pedersen (2023).

Here: what the estimated span *represents* in stock space, not a new estimator.

Alphas, factor compensation and identification

Zhang (2024); Choi and Yuan (2025); Feng, Giglio and Xiu (2020).

Here: attribution of a *fixed* fitted return across observed, latent and intercept components.

Characteristic portfolios and hedging

Brandt, Santa-Clara and Valkanov (2009); Daniel, Mota, Rottke and Santos (2020); Kelly, Malamud and Pedersen (2023); Jensen, Kelly, Malamud and Pedersen (2026).

Here: a *fixed* characteristic mandate, and the stocks, gross and risk it requires.

Setup: the stock-exposure ruler

Measure a loading by the stocks it loads on

Characteristics parameterize three components

$$r_{i,t+1} = \underbrace{z'_{i,t}\Gamma_\alpha}_{\text{characteristic intercept}} + \underbrace{z'_{i,t}C\, g_{t+1}}_{\text{observed factors (FFC6)}} + \underbrace{z'_{i,t}B\, f_{t+1}}_{\text{latent factors } (K=3)} + \varepsilon_{i,t+1}$$

- $z_{i,t}$: constant + centered ranks of **36** or **153** JKP characteristics.
- Library grows, **number of factors fixed**: C and B are re-estimated.
- Latent factors identified up to rotation, so the object is the **loading space**.

Three different value objects

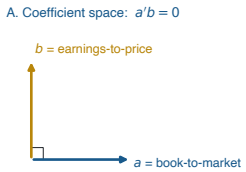
$Z_t a_{\mathrm{BM}}$	book-to-market profile (stocks)
$Z_t c_{\mathrm{HML}}$	estimated HML loading profile
HML_{t+1}	one monthly return

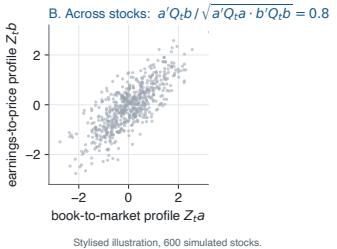
Their alignment across stocks must be *measured*.

Nested characteristic libraries can produce non-nested loading spaces.

The stock-exposure ruler

$$\frac{1}{n_t}\,(Z_t a)'(Z_t b) \;=\; a' Q_t\, b, \qquad Q_t = \frac{Z_t' Z_t}{n_t}$$





Orthogonal coefficients need not give orthogonal stock profiles:
$a' b = 0 \not\Rightarrow a' Q_t b = 0.$

Q_t already sits in the IPCA normal equations; I use it to *read* the estimates.

▸ why not return covariance?

Coverage: how much of a position the loading space represents

For a characteristic direction a, loadings $L = [C, B]$ and reference Q:

$$\mathcal{C}(a, L; Q) \;=\; \frac{a'QL\,(L'QL)^{-1}L'Qa}{a'Qa} \;\in [0, 1]$$

Reads as

Squared share of the stock profile $Z_t a$ that lies in the span of the loading profiles $Z_t L$.

Properties

- Invariant to latent rotation.
- Invariant to characteristic units.
- Not explained return variance.

Both libraries are scored on the same profiles under the same Q: the 36-characteristic loadings are embedded with zeros.

▸ units check

Data and design

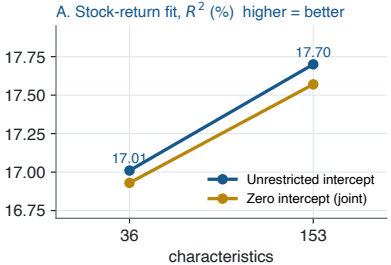
Stocks and signals	CRSP returns; 153 JKP characteristics vs a fixed 36-subset; monthly centered ranks
Model	FFC6 observed + 3 latent; characteristic intercept free *or* jointly zero
Historical comparison	July 1964–May 2014, 599 months, full-sample fits
Rolling evaluation	Re-estimate each March 2007–2025 on 240 months: 19 origins, 225 formation and 216 return months
Stock split	Fixed hash of security ID: source stocks learn and hold, target stocks test (about 2,250 each per month)
Mandates	Book-to-market, 11-month momentum, larger-firm size (not SMB)
Inference	12-month paired calendar blocks, 19,999 draws, simultaneous 95% intervals per family of 12

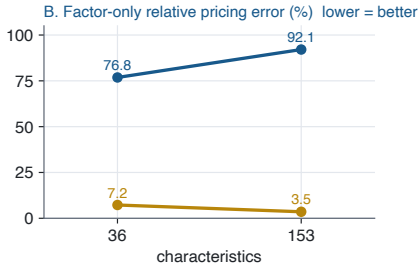
▸ samples ▸ measures

1. What the loading space represents

A richer library re-chooses a fixed-dimensional space

A first warning: better fit, worse factor pricing





Unrestricted intercept: the factor component explains 36 target means *worse* with 153 characteristics.

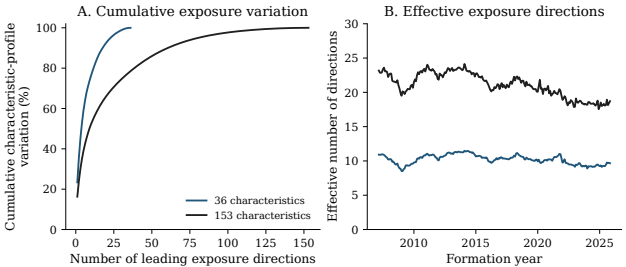
Intercept jointly set to zero: the ranking flips. The split between factors and intercept drives the comparison.

Return fit does not tell you what the factor component prices, or what it represents.

Historical sample 1964:07–2014:05, 599 months, 36 characteristic-managed targets.

▸ full table ▸ definitions

153 characteristics: more directions, and still much overlap



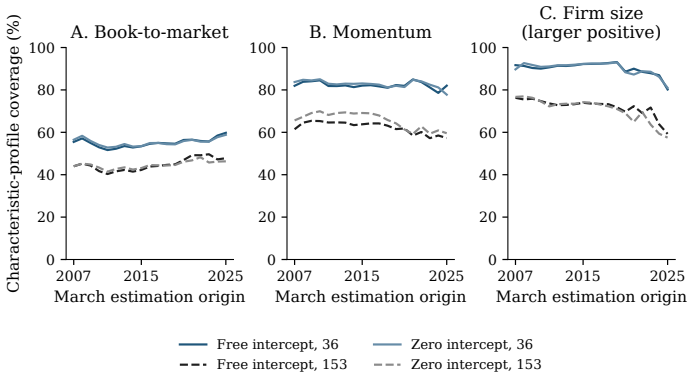
	36	153
Effective directions	10.22	21.08
Top-5 share	58.7%	38.8%

The 117 added profiles keep **51.3%** of their variation outside the original span.

Half new information, half re-description.

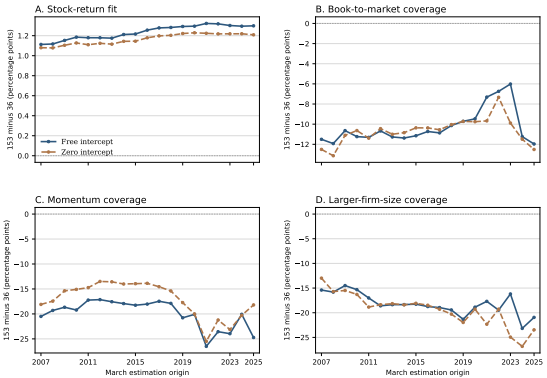
Characteristics are not independent signals: count stock-profile directions, not variables.

Coverage of value, momentum and size falls



Unrestricted, mean over 19 origins	Book-to-market	Momentum	Larger-firm size
36 → 153 characteristics	55.1 → **44.8**%	82.3 → **62.3**%	90.2 → **71.9**%

Fit up, coverage down: at every origin, in every design



153 minus 36; same 240-month window for fit and coverage.

Origin by origin: fit gain $+1.1$ to $+1.3$ pp; coverage loss in **19/19** origins for all three profiles.

Same direction with

- 120, 180, expanding windows
- $K = 2$ and $K = 5$ latent factors
- five random 36-baselines: **95/95**
- non-overlapping sub-periods

The decline is a property of the re-estimated space, not of one window or one baseline.

▸ robustness table ▸ non-overlapping

Not mechanical dilution: the space is re-chosen, not extended

The larger model uses new exposures

38.2% of the 153-characteristic loading space's exposure energy lies *outside* the original 36 and the constant.

Some groups gain

Intangibles and firm attributes: 35.6% → **37.6%**.

Familiar positions lose

Group coverage (%)	36	153
Value and payout	44.0	38.5
Momentum group	48.8	36.1
Profitability	32.9	23.9
Investment	18.8	14.0
Original 36 profiles, mean	49.9	35.4

With the factor count fixed, more characteristics buy new directions by giving up familiar ones.

▸ all 11 groups

2. Who gets the credit

Overlapping loadings make attribution a choice, with fit held fixed

Same fitted return, different credit

Within one fitted model, with $\bar{Q}$ the window-average ruler:

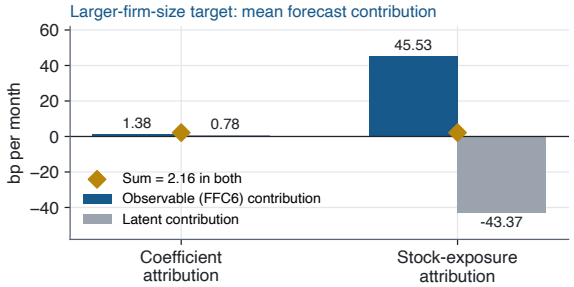
$$A = (B'\bar{Q}B)^{-1}B'\bar{Q}C$$

$$C_Q = C - BA, \qquad p^Q_{t+1} = p_{t+1} + A\,g_{t+1}$$

Then $B'\bar{Q}C_Q = 0$ and

$$Cg_{t+1} + Bp_{t+1} = C_Q g_{t+1} + Bp^Q_{t+1}.$$

Observed loadings lose what they share with the latent space on stocks; the latent payoff absorbs it.



The joint forecast stays at 2.16 bp. The observable share moves from 1.38 to 45.53 bp.

▸ all three targets

The intercept is a third claimant

Project the characteristic intercept onto the latent loading space under Q_t and move that part to the factor mean.

- Complete stock fit: **unchanged**.
- Factor-only pricing: **changes**, because it scores the factor share only.

Factor-only relative pricing error (%)

	36	153
Coefficient attr.	76.84	92.08
Stock-exposure attr.	**53.89**	**72.90**
Zero intercept, refit	7.25	3.55

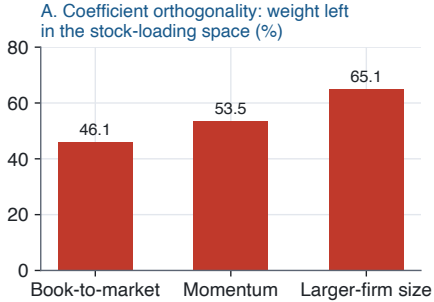
A factor-only pricing statistic scores a split, not a fit. Report which component gets the credit.

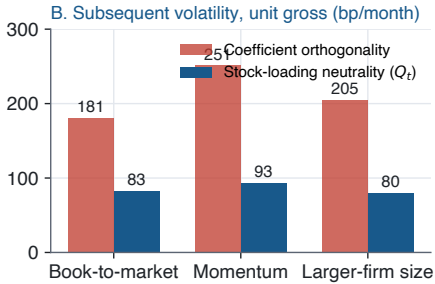
Unrestricted FFC6 + 3 latent, historical sample. Joint zero-intercept estimation is a different fitted model, not a reallocation.

3. Holding a fixed characteristic target

The loading space becomes stock positions

Coefficient orthogonality does not neutralize stock loadings



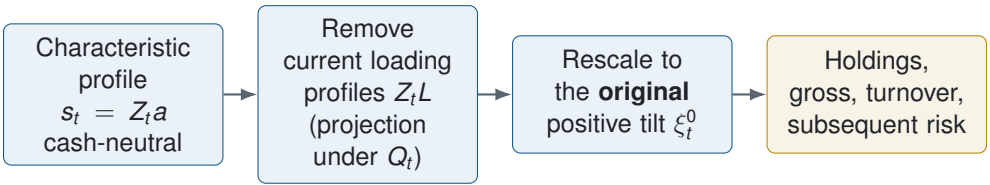


Projecting in coefficient space leaves 46–65% of the position on the estimated loading profiles.
The Q_t projection removes it to numerical precision.

Unrestricted 153-characteristic model, source stocks, unit gross, zero net investment; 225 formation months, 216 return months.

table

Experiment: keep the investor's target, change the loading space

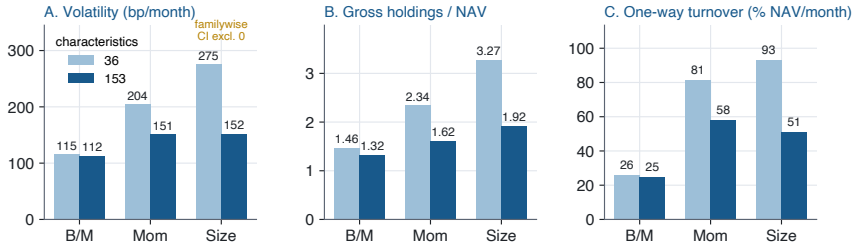


$$\widetilde{w}_t = \frac{Z_t}{n_t}\Big[a - L(L'Q_tL)^{-1}L'Q_ta\Big], \qquad L = [e_0, C, B], \qquad w_t = \frac{\xi_t^0}{w_t^{N\prime}s_t}\, w_t^N$$

Compare: 36 vs 153 loading spaces, and a historical minimum-variance hedge at the same target (Daniel, Mota, Rottke and Santos, 2020).

Fixed: stocks, characteristic, target, cash neutrality.
Outcomes: what it takes to deliver the same bet.

Same target, different library: holdings and risk



Larger-firm size: volatility -123 bp $[-167, -80]$, gross $-1.35\times$NAV, turnover -42 pp.

3 of 12 simultaneous intervals exclude zero: size (both intercepts) and zero-intercept momentum.

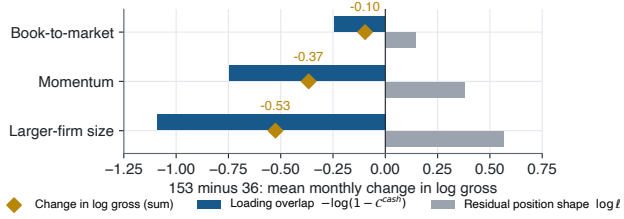
Same characteristic bet, very different positions. The risk effect is large where coverage fell most.

Current-Q loading neutrality, unrestricted intercept. Brackets: simultaneous 95% interval, twelve-comparison family.

▸ all contrasts ▸ vs covariance hedge

Why gross changes: an exact identity

$$G_t = \frac{\xi_t^0}{\eta_t} = \frac{\ell_t}{1 - \mathcal{C}_t^{\text{cash}}}, \qquad \log G_t = \underbrace{-\log\left(1 - \mathcal{C}_t^{\text{cash}}\right)}_{\text{loading overlap}} + \underbrace{\log \ell_t}_{\text{residual shape}}$$



$\mathcal{C}^{\text{cash}}$: share of the profile the hedge removes.

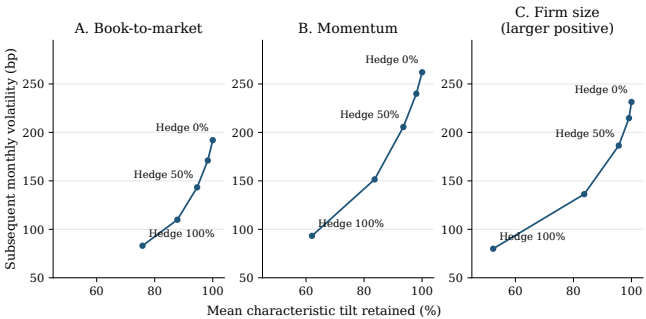
ℓ: absolute size of what is left, relative to the original.

Size: overlap contributes $-\mathbf{1.09}$, shape $+0.57$, net -0.53 in log gross.

Lower coverage leaves more of the bet outside the hedge, so less gross is needed to restore it.

table

How much to hedge: tilt kept versus risk removed



Book-to-market	tilt kept	vol (bp)
Unhedged	100.0%	192
50% hedge	94.6%	143
Full hedge	75.8%	83

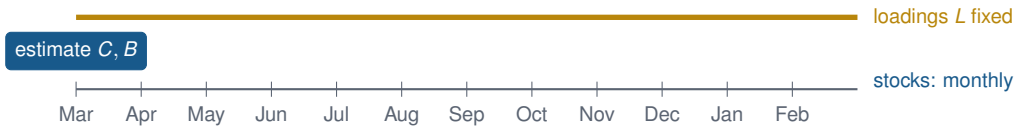
Full hedge keeps only **62%** of momentum and **52%** of size at unit gross.

Unit gross costs tilt; the original tilt costs gross. The choice depends on the investor.

4. The ruler moves

Fixed coefficients, changing stocks: which cross section to measure on?

Coefficients fixed for a year; stocks change every month



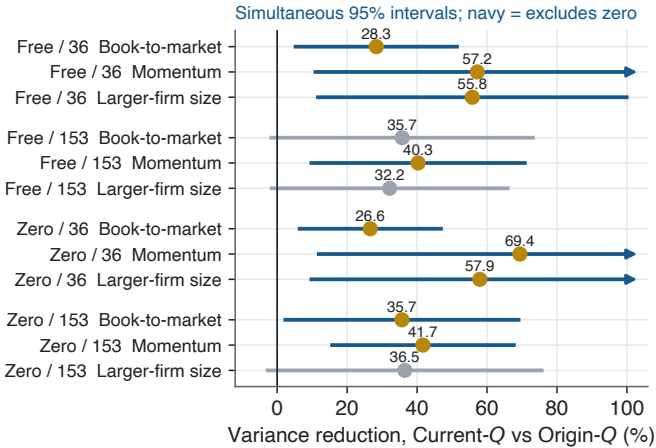
Training-Q average of the 240 training cross sections, fixed for the year

Origin-Q the March cross section, fixed for the year

Current-Q this month's stocks and characteristics, Q_t

Same L, same current stocks, same original positive target, same cash neutrality. Only Current-Q is neutral to the loadings of the stocks actually held.

Current measurement: lower risk at the same target



All 12 point estimates favor Current-Q: **26.6–69.4%** lower variance.

9 of 12 simultaneous intervals exclude zero.

Signed-tilt family: **8 of 12**.

Gap narrows with more frequent refits; same direction on stocks never used in estimation.

▸ table ▸ signed family ▸ design sensitivity ▸ returns, gross

When a stale ruler explodes the position

Zero-intercept 36-characteristic model, larger-firm size, **June 2008**. Same stocks, same target exposure 0.3355.

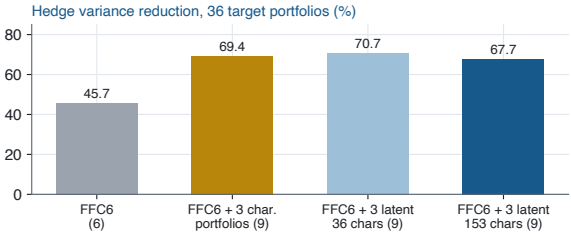
	Training-Q	Origin-Q	Current-Q
Target exposure per unit gross	0.000039	0.084	0.085
Gross needed / NAV	**8,607.78**	3.99	3.94
Alignment with current loadings	0.51	0.27	2.5×10^{-14}

Across all months, maximum gross: Origin-Q **12.38**, Current-Q **4.09** $\times$ NAV.

If the hedge direction keeps almost none of the characteristic, restoring the bet needs extreme gross.
Exposure matching and the holdings it requires must be read together.

▸ full case

What latent payoffs add, out of sample



Factor-only pricing	RPE (%)	RMS (bp)
FFC6	61.53	20.33
+ latent, 36	56.02	19.40
+ latent, 153	50.79	18.47

Adding latent to the 9-instrument menu: $+1.81$ pp [0.63, 3.25] with 36 characteristics.

Size target: FFC6 hedges 13.7%; with three characteristic portfolios 86.3%.

Latent payoffs hedge well, but three characteristic portfolios the investor can already build do almost as well.

Separate target stocks; slopes from past data; 216 common months.

▸ pricing and forecasts ▸ paired intervals

Conclusion

1. **Measure loadings by their stocks.** The cross-sectional second moment Q_t tells you what a loading space represents; fit and coefficient values do not.
2. **More characteristics re-choose the space.** Fit rises while value, momentum and size coverage falls, at every origin.
3. **Overlap is an attribution choice.** The same fitted return supports very different credit to observed, latent and intercept components.
4. **Overlap becomes holdings.** The same characteristic target needs different stocks, gross and risk; the current ruler lowers risk at that target.

Evaluating a characteristic model means four linked objects:
the investor's target, the loading rule, the stocks it requires, and the returns they earn.

Thank you

Mingyang Liu • yang.liu19@imperial.ac.uk

The ruler matters: for what a model represents, who gets the credit, and what you hold.

Backup map

Setup
B1 Samples
B2 Performance measures
B3 Why Q_t, not return covariance
B4 Units check

Representation
B5 Fit and pricing table
B6 Fit vs coverage: designs
B7 Non-overlapping samples
B8 Coverage by group

Attribution
B9 Three targets

Holdings
B10 Coefficient vs stock neutrality
B11 Library contrasts with CIs
B12 vs minimum-variance hedge
B13 Gross identity table

The ruler moves
B14 Positive-target family
B15 Signed-target family
B16 Design sensitivity
B17 Returns, gross, turnover
B18 June 2008 case

Payoffs
B19 Pricing and forecasts
B20 Hedge menus
B21 Changing conditions

B1 Samples and characteristic groups

Economic group	36-characteristic library	153-characteristic library
Accounting quality	2	17
Composite signals	2	7
Financing and leverage	4	15
Intangibles and firm attributes	1	9
Investment	4	23
Trading liquidity	3	9
Momentum	5	10
Profitability	6	22
Historical return risk	4	18
Seasonality	1	10
Value and payout	4	13

Experiment or population	Calendar or statistic	Sample size or summary
Historical comparison	July 1964–May 2014	599 formation months
Rolling evaluation	March 2007–November 2025	225 formation months
Common payoff evaluation	Same rolling calendar	216 return months
Model estimation	Each March, 2007–2025	19 origins; 240 prior months
Source stocks per formation month	Mean / minimum / maximum	2250 / 2018 / 2744
Target stocks per formation month	Mean / minimum / maximum	2265 / 2042 / 2735

Ranks are centered at one-half; missing values get zero. A fixed hash of the CRSP identifier splits source and target stocks for the whole sample.

back

B2 Performance measures

Test payoff $y_{j,t+1}$; factor component $q^F_{j,t+1} = b'_{j,t} F_{t+1}$; target-regression intercept $c_{j,t}$ kept out of the factor-only statistics.

$$R^2_{\text{total}} = 1 - \frac{\sum_{j,t}(y_{j,t+1} - q^F_{j,t+1})^2}{\sum_{j,t} y^2_{j,t+1}}, \qquad \text{RPE} = \frac{\sum_j (\alpha^{\text{PE}}_j)^2}{\sum_j \bar{y}_j{}^2}, \qquad \alpha^{\text{PE}}_j = \tfrac{1}{T}\sum_t (y_{j,t+1} - q^F_{j,t+1})$$

$$R^2_{\text{pred}} = 1 - \frac{\sum_{j,t}(y_{j,t+1} - b'_{j,t}\hat{\lambda}_t)^2}{\sum_{j,t} y^2_{j,t+1}}, \qquad \text{HVR} = 1 - \frac{\sum_j \text{Var}_t(y_{j,t+1} - b'_{j,t}F_{t+1})}{\sum_j \text{Var}_t(y_{j,t+1})}$$

$$\text{Variance reduction} = 1 - \frac{\text{Var}(r^{\text{treatment}})}{\text{Var}(r^{\text{reference}})}, \qquad \text{gross} = \|w_t\|_1, \qquad \text{turnover} = \tfrac{1}{2}\sum_i |w_{i,t} - w^{\text{pre}}_{i,t}|$$

Historical comparison: complete stock-return R^2 with equal month weights; RPE on 36 unscaled characteristic-managed targets.

◂ data ◂ puzzle ◂ payoffs

B3 Why the characteristic second moment, not return covariance?

Different questions

Q_t: do two positions load on the *same stocks*?

Σ: do two positions *move together*?

A loading restriction is imposed on stocks held at formation, so it is stated in Q_t.

Not a new estimator

Q_t already enters the IPCA normal equations. Estimation is unchanged; only the reading of the estimates uses it.

Rotation invariance

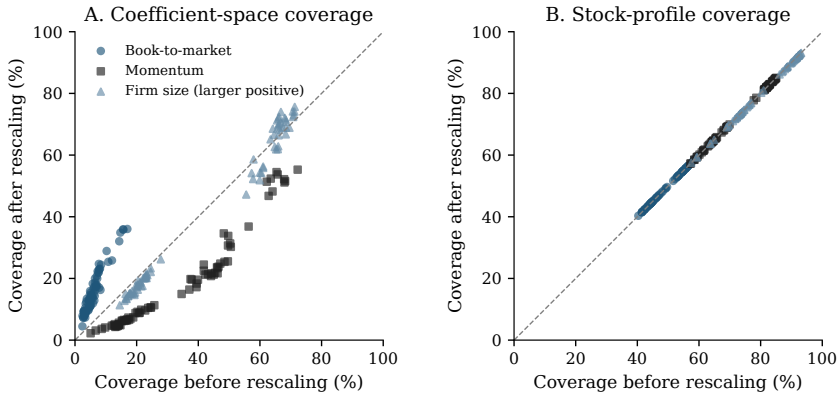
Coverage and loading neutrality depend on the span of $Z_t L$, so any admissible change of latent basis leaves them unchanged.

The covariance alternative is in the paper

Historical minimum-variance hedge at the same target and span (Daniel et al., 2020). Rankings differ by mandate (covariance vs loading-neutral volatility): 36-size 185 vs 275 bp; 153-momentum 229 vs 151 bp.

back B12

B4 Changing units, keeping every stock exposure



Characteristics rescaled by $[\frac{1}{2}, 2]$, coefficients inversely: stock profiles unchanged. Coefficient coverage moves (A); stock-profile coverage does not (B).

back

B5 Historical fit and factor-only pricing

Factor model	Source characteristic intercept	Characteristics	Complete stock-return R^2 (%)	Factor-only relative pricing error (%)	
				Coefficient attribution	Stock-exposure attribution
3 latent + FFC6	Unrestricted	36	17.01	76.84	53.89
3 latent + FFC6	Unrestricted	153	17.70	92.08	72.90
3 latent + FFC6	Zero (joint)	36	16.93	7.25	7.25
3 latent + FFC6	Zero (joint)	153	17.57	3.55	3.55
5 latent	Unrestricted	36	17.06	70.93	43.16
5 latent	Unrestricted	153	17.71	127.64	94.49
5 latent	Zero (joint)	36	17.01	1.42	1.42
5 latent	Zero (joint)	153	17.62	0.44	0.44

Five-latent models show the same pattern: unrestricted RPE 70.9 $\rightarrow$ 127.6%; zero intercept 1.42 $\rightarrow$ 0.44%. RPE above 100% means worse than a zero forecast of target means, not negative returns.

B6 Fit versus coverage across designs (153 minus 36)

		Training R^2 (%)			Book-to-market (pp)		Momentum (pp)		Larger-firm size (pp)	
Design element	Intercept	36	153	Fit > 0	Mean [smallest]	< 0	Mean [smallest]	< 0	Mean [smallest]	< 0
Training window: 120 months	Free	15.90	17.75	19/19	-12.53 [-7.71]	19/19	-23.09 [-18.31]	19/19	-23.82 [-18.32]	19/19
	Zero	15.80	17.53	19/19	-13.85 [-10.15]	19/19	-22.04 [-18.62]	19/19	-25.27 [-16.82]	19/19
Training window: 180 months	Free	15.93	17.39	19/19	-11.09 [-6.67]	19/19	-21.02 [-15.96]	19/19	-20.62 [-14.31]	19/19
	Zero	15.83	17.20	19/19	-12.06 [-9.61]	19/19	-19.40 [-15.43]	19/19	-22.38 [-16.48]	19/19
Training window: 240 months, $K = 3$ (paper)	Free	15.46	16.69	19/19	-10.28 [-6.02]	19/19	-19.94 [-17.14]	19/19	-18.23 [-14.49]	19/19
	Zero	15.35	16.52	19/19	-10.67 [-7.33]	19/19	-17.14 [-13.49]	19/19	-19.39 [-12.99]	19/19
Training window: expanding (276–492 months)	Free	14.10	15.06	19/19	-10.36 [-8.13]	19/19	-17.84 [-16.61]	19/19	-13.95 [-12.83]	19/19
	Zero	14.00	14.91	19/19	-9.59 [-7.12]	19/19	-15.14 [-13.18]	19/19	-14.42 [-12.72]	19/19
Latent factors: $K = 2$	Free	15.19	16.31	19/19	-11.42 [-8.51]	19/19	-21.27 [-15.29]	19/19	-17.40 [-14.23]	19/19
	Zero	15.07	16.12	19/19	-11.15 [-9.68]	19/19	-17.22 [-14.84]	19/19	-19.10 [-17.61]	19/19
Latent factors: $K = 5$	Free	15.86	17.30	19/19	-11.17 [-6.81]	19/19	-18.69 [-14.15]	19/19	-17.67 [-15.56]	19/19
	Zero	15.79	17.16	19/19	-9.25 [-6.40]	19/19	-17.81 [-12.56]	19/19	-17.31 [-13.58]	19/19
Baseline library: five random 36-characteristic draws (pooled)	Free	15.06	16.69	95/95	-14.96 [-6.84]	95/95	-26.86 [-19.72]	95/95	-18.30 [-11.60]	95/95
	Zero	14.96	16.52	95/95	-14.61 [-4.55]	95/95	-23.01 [-15.71]	95/95	-19.89 [-12.11]	95/95

Entries: mean change in pp, smallest absolute change in brackets, and the count of origins with the stated sign.

back

B7 Non-overlapping and extended samples

				Stock-return R^2 (%)		Book-to-market (%)		Momentum (%)		Larger-firm size (%)	
Sample	Formation months	T	Intercept	36	153	36	153	36	153	36	153
Historical, non-overlapping: First half	1964-07 to 1989-06	300	Free	21.30	22.03	74.68	55.69	87.63	78.03	93.84	86.68
Historical, non-overlapping: First half	1964-07 to 1989-06	300	Zero	21.22	21.91	76.04	57.63	88.66	78.01	95.65	88.72
Historical, non-overlapping: Second half	1989-07 to 2014-05	299	Free	14.73	15.64	56.34	45.85	82.70	66.28	91.38	79.43
Historical, non-overlapping: Second half	1989-07 to 2014-05	299	Zero	14.60	15.49	59.02	50.97	82.23	68.22	92.91	78.61
Historical, non-overlapping: First third	1964-07 to 1981-02	200	Free	27.72	28.65	74.65	57.43	85.41	71.63	93.51	85.75
Historical, non-overlapping: First third	1964-07 to 1981-02	200	Zero	27.60	28.48	74.19	58.34	85.71	77.03	95.80	86.53
Historical, non-overlapping: Second third	1981-03 to 1997-10	200	Free	10.45	11.26	63.00	42.06	86.75	65.30	91.44	83.08
Historical, non-overlapping: Second third	1981-03 to 1997-10	200	Zero	10.39	11.14	68.50	52.48	87.34	68.12	92.61	84.25
Historical, non-overlapping: Third third	1997-11 to 2014-05	199	Free	17.92	19.02	54.33	44.23	81.54	64.74	91.78	75.15
Historical, non-overlapping: Third third	1997-11 to 2014-05	199	Zero	17.81	18.85	54.72	44.88	82.89	67.41	91.83	73.84
Historical, extended: Full history	1964-07 to 2025-11	737	Free	14.76	15.35	64.16	54.22	88.06	74.08	95.54	87.31
Historical, extended: Full history	1964-07 to 2025-11	737	Zero	14.70	15.26	64.90	55.44	89.13	77.94	95.40	85.78
Historical, extended: Post-2014 only	2014-06 to 2025-11	138	Free	9.91	10.90	57.47	43.87	81.21	53.61	87.73	65.99
Historical, extended: Post-2014 only	2014-06 to 2025-11	138	Zero	9.86	10.75	57.32	44.98	78.21	55.59	83.77	61.56
Rolling inputs, non-overlapping: 120-month window to 199403	1984-03 to 1994-02	120	Free	10.77	12.14	55.37	37.12	79.67	53.37	87.81	70.48
Rolling inputs, non-overlapping: 120-month window to 199403	1984-03 to 1994-02	120	Zero	10.65	11.93	58.89	43.04	77.95	55.92	88.08	69.82
Rolling inputs, non-overlapping: 120-month window to 200403	1994-03 to 2004-02	120	Free	16.63	18.16	57.05	43.60	72.99	50.11	91.88	70.60
Rolling inputs, non-overlapping: 120-month window to 200403	1994-03 to 2004-02	120	Zero	16.48	17.94	56.19	41.89	79.60	60.55	91.87	70.07
Rolling inputs, non-overlapping: 120-month window to 201403	2004-03 to 2014-02	120	Free	16.78	18.66	43.71	35.23	73.99	54.33	79.50	58.67
Rolling inputs, non-overlapping: 120-month window to 201403	2004-03 to 2014-02	120	Zero	16.67	18.41	45.61	34.59	80.92	58.19	79.60	57.15
Rolling inputs, non-overlapping: 120-month window to 202403	2014-03 to 2024-02	120	Free	13.00	14.78	62.12	47.46	66.82	43.86	82.89	56.06
Rolling inputs, non-overlapping: 120-month window to 202403	2014-03 to 2024-02	120	Zero	12.95	14.60	61.28	46.44	67.60	46.14	81.19	54.30
Rolling inputs, non-overlapping: 240-month window to 200403	1984-03 to 2004-02	240	Free	13.89	14.99	59.86	46.36	81.93	61.37	90.93	75.93
Rolling inputs, non-overlapping: 240-month window to 200403	1984-03 to 2004-02	240	Zero	13.77	14.84	61.22	47.06	83.73	66.66	89.64	76.11
Rolling inputs, non-overlapping: 240-month window to 202403	2004-03 to 2024-02	240	Free	14.18	15.48	58.46	47.22	78.58	58.50	86.94	63.77
Rolling inputs, non-overlapping: 240-month window to 202403	2004-03 to 2024-02	240	Zero	14.12	15.34	57.69	46.20	81.18	60.90	86.16	59.35

back

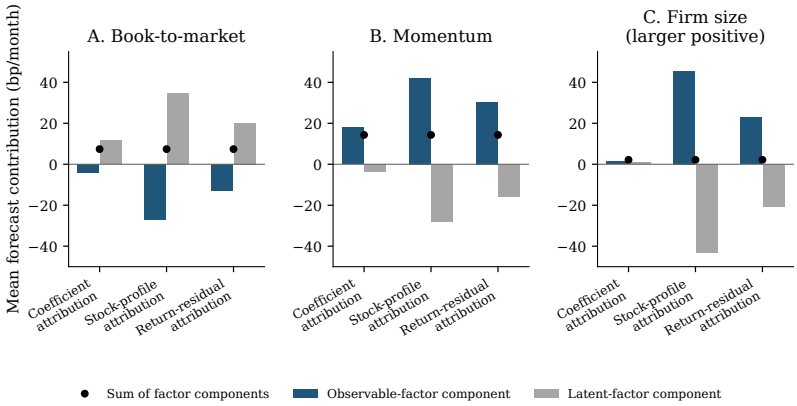
B8 Joint loading-space coverage by economic group

Economic group	Characteristics in full library	Joint loading-space coverage (%)			
		Free source intercept		Zero source intercept	
		36	153	36	153
Accounting quality	17	13.38	10.30	13.41	10.58
Composite signals	7	31.53	22.74	33.32	25.37
Financing and leverage	15	27.19	22.20	28.39	23.60
Intangibles and firm attributes	9	35.62	37.59	35.88	37.58
Investment	23	18.84	13.96	18.64	13.56
Trading liquidity	9	57.36	45.18	57.17	43.36
Momentum	10	48.80	36.14	48.66	37.89
Profitability	22	32.87	23.88	33.49	25.43
Historical return risk	18	33.75	27.62	34.34	28.72
Seasonality	10	9.51	9.26	9.49	9.35
Value and payout	13	44.00	38.51	44.41	39.09

Across the original 36 profiles mean coverage falls 49.94 $\rightarrow$ 35.38%; across the added 117, 24.05 $\rightarrow$ 20.83%.

back

B9 Attribution for the three economic targets



Unrestricted 36-characteristic model. Diamonds: the joint factor forecast, identical across attributions. Return-residual attribution regresses latent payoffs on FFC6 using past data only.

back

B10 Coefficient orthogonality vs stock-loading neutrality

Characteristic mandate	Exposure restriction	Loading-projection share (%)	Mean excess return (bp/month)	Return volatility (bp/month)	One-way turnover (% NAV/month)
Book-to-market	Coefficient orthogonality	46.10	24.49	181.03	13.05
Book-to-market	Stock-loading neutrality	0.00	12.93	83.09	18.40
Momentum	Coefficient orthogonality	53.51	42.99	251.31	23.81
Momentum	Stock-loading neutrality	0.00	21.91	93.30	35.71
Firm size	Coefficient orthogonality	65.13	9.23	205.45	9.87
Firm size	Stock-loading neutrality	0.00	-8.54	79.99	26.52

Unit gross for both rules; equal gross does not hold the characteristic exposure fixed.

back

B11 Library expansion at the same positive target: 153 minus 36

	Mean excess return change (bp/month) Pointwise CI	Volatility change (bp/month) Familywise CI	Mean gross change (NAV units) Pointwise CI	One-way turnover change (pp/month) Pointwise CI
Free source characteristic intercept				
Book-to-market	-1.29 [-10.56,9.46]	-2.82 [-27.67,22.02]	-0.135 [-0.151,-0.119]	-1.32 [-2.01,-0.69]
Momentum	16.12 [-7.18,36.28]	-53.00 [-107.19,1.19]	-0.718 [-0.764,-0.676]	-23.57 [-25.71,-21.57]
Larger-firm size	-28.52 [-62.40,7.36]	-123.18 [-166.54,-79.81]	-1.349 [-1.479,-1.223]	-41.70 [-46.02,-37.23]
Zero source characteristic intercept				
Book-to-market	-1.44 [-11.21,10.26]	-5.85 [-27.37,15.66]	-0.144 [-0.167,-0.124]	-3.36 [-4.30,-2.51]
Momentum	23.13 [1.57,45.36]	-55.04 [-99.43,-10.65]	-0.714 [-0.776,-0.656]	-23.79 [-25.24,-22.23]
Larger-firm size	-42.60 [-77.92,-6.49]	-120.58 [-172.92,-68.23]	-1.372 [-1.493,-1.248]	-42.39 [-47.76,-36.94]

Volatility: simultaneous 95% intervals (family of 12). Others: pointwise.

back

B12 Loading neutrality versus historical minimum variance

Source intercept / no. of characteristics	Characteristic mandate	Historical minimum variance			Current-Q loading neutrality		
		Volatility (bp/month)	Gross holdings/NAV	One-way turnover (% NAV/month)	Volatility (bp/month)	Gross holdings/NAV	One-way turnover (% NAV/month)
Unrestricted / 36	Book-to-market	131.53	1.23	18.86	115.29	1.46	25.89
Unrestricted / 36	Momentum	240.14	1.42	45.31	204.26	2.34	81.40
Unrestricted / 36	Firm size	185.25	1.35	23.57	275.30	3.27	92.99
Unrestricted / 153	Book-to-market	141.18	1.21	20.03	112.47	1.32	24.56
Unrestricted / 153	Momentum	228.99	1.33	42.44	151.25	1.62	57.84
Unrestricted / 153	Firm size	171.25	1.28	24.86	152.12	1.92	51.29
Zero / 36	Book-to-market	133.94	1.24	19.14	115.09	1.47	27.62
Zero / 36	Momentum	252.19	1.42	44.15	213.32	2.42	83.17
Zero / 36	Firm size	179.75	1.34	22.57	273.32	3.27	91.69
Zero / 153	Book-to-market	142.76	1.21	19.73	109.24	1.32	24.26
Zero / 153	Momentum	228.61	1.33	42.97	158.28	1.70	59.38
Zero / 153	Firm size	172.05	1.27	24.55	152.74	1.89	49.30

Same target, same span. Minimum variance optimizes historical payoff variance; loading neutrality removes current loadings. All six covariance-rule library intervals include zero.

back B3

B13 Gross identity: monthly accounting

Source intercept	Characteristic	Cash-adjusted coverage: 36 (%)	Cash-adjusted coverage: 153 (%)	Loading overlap contribution	Residual stock shape contribution	Log gross 153 minus 36 change
Free	Book-to-market	58.71	47.38	-0.2431	0.1464	-0.0967
Free	Momentum	83.48	65.14	-0.7455	0.3791	-0.3665
Free	Larger-firm size	91.84	76.27	-1.0926	0.5674	-0.5252
Zero	Book-to-market	59.08	47.43	-0.2531	0.1502	-0.1028
Zero	Momentum	84.60	68.47	-0.7206	0.3717	-0.3490
Zero	Larger-firm size	91.72	75.22	-1.1189	0.5797	-0.5392

Averages of monthly 153-minus-36 differences; the two contributions sum exactly to the log-gross change along the selected projection direction.

back

B14 Current vs Origin at the original positive target

Source intercept / no. of characteristics	Characteristic	Variance reduction vs. Origin (%)	Familywise 95% CI (%)	Adjusted p
Free / 36	Book-to-market	28.34	[5.28, 51.40]	0.015
Free / 36	Momentum	57.22	[10.96, 103.49]	0.014
Free / 36	Larger-firm size	55.81	[11.68, 99.93]	0.012
Free / 153	Book-to-market	35.74	[-1.64, 73.11]	0.063
Free / 153	Momentum	40.29	[9.78, 70.81]	0.009
Free / 153	Larger-firm size	32.17	[-1.58, 65.91]	0.063
Zero / 36	Book-to-market	26.62	[6.42, 46.83]	0.009
Zero / 36	Momentum	69.39	[11.91, 126.87]	0.016
Zero / 36	Larger-firm size	57.94	[9.79, 106.09]	0.016
Zero / 153	Book-to-market	35.67	[2.33, 69.01]	0.035
Zero / 153	Momentum	41.69	[15.72, 67.66]	0.003
Zero / 153	Larger-firm size	36.48	[-2.68, 75.64]	0.071

19,999 twelve-month circular block draws, studentized max-$|t|$. Bonferroni percentile bands: 12 of 12; studentized log variance ratio: 0 of 12.

back

B15 Current vs Origin at the training rule's signed tilt

Source intercept / no. of characteristics	Characteristic	Variance reduction vs. Origin (%)	Familywise 95% CI (%)	Adjusted p
Free / 36	Book-to-market	27.18	[9.08, 45.27]	0.004
Free / 36	Momentum	44.45	[12.98, 75.93]	0.007
Free / 36	Larger-firm size	28.36	[-19.68, 76.40]	0.335
Free / 153	Book-to-market	32.40	[4.91, 59.89]	0.021
Free / 153	Momentum	41.97	[16.58, 67.35]	0.002
Free / 153	Larger-firm size	18.61	[-8.38, 45.59]	0.216
Zero / 36	Book-to-market	25.80	[9.47, 42.14]	0.003
Zero / 36	Momentum	45.44	[18.45, 72.42]	0.002
Zero / 36	Larger-firm size	33.01	[-10.75, 76.77]	0.161
Zero / 153	Book-to-market	33.81	[7.80, 59.82]	0.012
Zero / 153	Momentum	42.37	[20.54, 64.20]	0.001
Zero / 153	Larger-firm size	22.80	[-3.77, 49.38]	0.099

Separate family of twelve; all twelve point estimates favor Current-Q.

back

B16 Design sensitivity of the main comparisons

		Positive target: Current vs. Origin (12)			Signed target: Current vs. Origin (12)			Library expansion, loading neutral (6)	
Design	Variant	Range of estimates (%)	Studentized	Percentile	Range of estimates (%)	Studentized	Percentile	Range (bp/month)	Studentized / percentile
Re-estimation calendar	Annual, March (paper)	26.6 to 69.4	9/12	12/12	18.6 to 45.4	8/12	10/12	-123.2 to -2.8	4/6 / 4/6
Re-estimation calendar	Semiannual, March and September	18.7 to 85.7	1/12	12/12	11.7 to 59.6	1/12	8/12	-129.8 to -4.5	4/6 / 4/6
Re-estimation calendar	Quarterly	6.6 to 63.1	0/12	7/12	6.3 to 34.4	3/12	6/12	-125.2 to -4.2	4/6 / 4/6
Training window	120 months	22.6 to 60.3	8/12	12/12	24.1 to 51.6	8/12	12/12	-83.3 to -1.7	4/6 / 4/6
Training window	180 months	26.8 to 57.6	12/12	12/12	24.7 to 46.4	8/12	12/12	-75.4 to -5.9	4/6 / 4/6
Training window	Expanding	29.4 to 99.1	3/12	10/12	21.1 to 99.4	3/12	11/12	-163.4 to -5.9	2/6 / 4/6
Latent factors	$K = 2$	26.2 to 67.2	8/12	12/12	23.1 to 46.8	10/12	12/12	-105.0 to -6.5	4/6 / 4/6
Latent factors	$K = 5$	32.5 to 78.5	5/12	12/12	26.6 to 76.6	2/12	11/12	-147.5 to -4.0	2/6 / 4/6
Stock universe	Target stocks (not used in estimation)	19.9 to 81.3	4/12	7/12	10.0 to 51.2	6/12	6/12	-91.5 to -13.3	5/6 / 6/6

Counts: intervals excluding zero. More frequent refits shrink the gap between Origin and Current, as expected when the reference is refreshed more often.

back

B17 Positive target: returns, gross and turnover

Source intercept / no. of characteristics	Characteristic	Rule	Mean excess return (bp/month)	Volatility (bp/month)	Median gross/NAV (multiple)	Maximum gross/NAV (multiple)	One-way turnover fraction of NAV per month
Free / 36	Book-to-market	Origin	18.14	136.19	1.460	1.972	0.268
		Current	18.32	115.29	1.437	1.733	0.259
Free / 36	Momentum	Origin	25.30	312.30	2.244	8.861	0.873
		Current	18.30	204.26	2.306	3.031	0.814
Free / 36	Larger-firm size	Origin	-15.26	414.13	3.554	8.884	1.128
		Current	12.85	275.30	3.334	3.958	0.930
Free / 153	Book-to-market	Origin	12.34	140.30	1.319	1.839	0.253
		Current	17.03	112.47	1.314	1.504	0.246
Free / 153	Momentum	Origin	32.76	195.75	1.579	2.742	0.575
		Current	34.42	151.25	1.587	2.242	0.578
Free / 153	Larger-firm size	Origin	-20.61	184.70	1.929	3.276	0.545
		Current	-15.66	152.12	1.921	2.286	0.513
Zero / 36	Book-to-market	Origin	7.87	134.36	1.476	1.905	0.288
		Current	6.23	115.09	1.450	1.764	0.276
Zero / 36	Momentum	Origin	-15.58	385.57	2.314	12.380	0.923
		Current	-24.74	213.32	2.394	3.137	0.832
Zero / 36	Larger-firm size	Origin	-0.29	421.43	3.447	11.679	1.089
		Current	31.60	273.32	3.404	4.094	0.917
Zero / 153	Book-to-market	Origin	1.85	136.20	1.325	1.730	0.248
		Current	4.79	109.24	1.309	1.486	0.243
Zero / 153	Momentum	Origin	-5.11	207.27	1.693	3.224	0.597
		Current	-1.60	158.28	1.702	2.300	0.594
Zero / 153	Larger-firm size	Origin	-18.51	191.64	1.931	3.454	0.529
		Current	-11.00	152.74	1.919	2.260	0.493

back

B18 June 2008: an exposure-cancelling direction

Quantity	Training Q	Origin Q	Current Q
Positive characteristic target	0.33549791	0.33549791	0.33549791
Unit-gross remaining exposure	0.00003898	0.08413278	0.08505412
Gross holdings / NAV	8607.78	3.99	3.94
Current loading alignment (L^2 norm ratio)	0.508481	0.274707	2.52×10^{-14}
Reference Gram condition number	289.95	357.89	374.88
Next-period model wealth factor	-90.207438	0.973842	1.008599

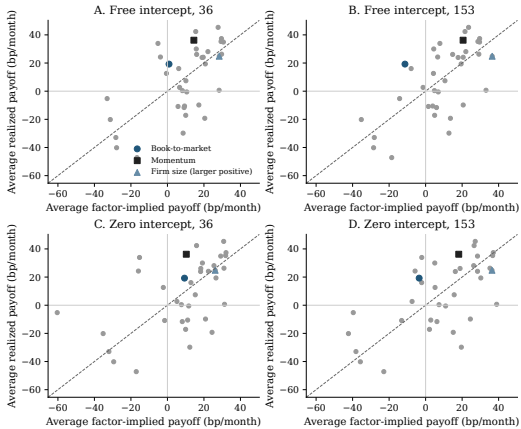
The 8,607.78 comes from scaling this training-reference direction, not from minimizing gross over feasible portfolios.

back

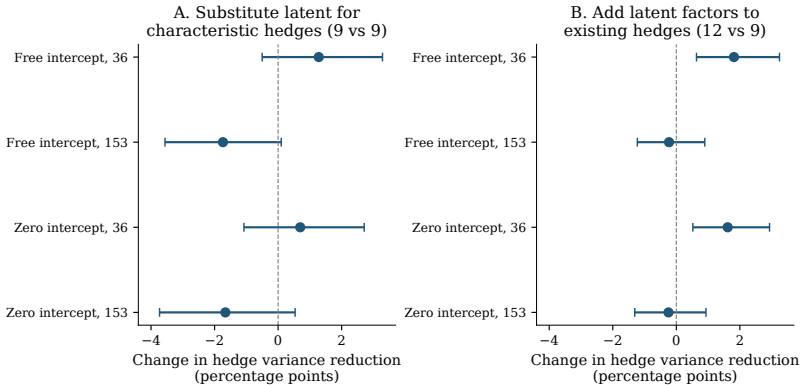
B19 Out-of-sample pricing and forecasts

	Factor-only realized-payoff fit and pricing			Predictive performance	
Factor-payoff model	Total R^2 (%)	Relative pricing error (%)	RMS pricing error (bp/month)	Factor-only R^2 (%)	Complete R^2 (%)
FFC6	45.53	61.53	20.33	0.16	0.20
Unrestricted / 36	70.25	56.02	19.40	0.02	0.50
Unrestricted / 153	67.38	50.79	18.47	0.32	0.47
Zero / 36	69.36	74.85	22.43	-0.34	0.51
Zero / 153	67.26	61.76	20.37	0.22	0.48

Predictive R^2 is small and positive against zero; complete forecasts are slightly negative against the historical mean. The 153-minus-36 RMS difference has a paired interval covering zero.



B20 Hedge menus: substitution and addition



Replacement (9 vs 9): all intervals include zero. Addition (12 vs 9): 36-characteristic latent adds $+1.81$ pp [0.63, 3.25]; 153 adds -0.23 pp.

back

B21 Changing market conditions

Source intercept / no. of characteristics	Mean absolute coverage contribution (pp)		Latent-subspace distance (%)
	Loadings	Characteristic distribution	
Unrestricted / 36	1.92	0.27	6.08
Unrestricted / 153	1.08	0.24	8.88
Zero / 36	1.61	0.29	6.09
Zero / 153	1.39	0.24	13.89

Coverage changes across origins come mostly from re-estimated loadings, less from the characteristic distribution. Exposure drift does not forecast next month's squared hedge loss once past volatility and loss are controlled.

back